Generalized Compare-and-Swap and Space-Efficient Universal Constructions for the Infinite-Arrival Model

Vassos Hadzilacos, **Myles Thiessen**, Sam Toueg

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Compare-and-Swap Object O

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CAS(val1, val2)

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```
CAS(val1, val2)
  current := O
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Compare-and-Swap Object O

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CAS(val1, val2)
  current := O
  if current = val1 then
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Compare-and-Swap Object O

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CAS(val1, val2)
  current := O
  if current = val1 then
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Compare-and-Swap Object O

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CAS(val1, val2)
  current := O
  if current = val1 then
    O := val2
    return True
```

Compare-and-Swap Object O

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CAS(val1, val2)
  current := O
  if current = val1 then
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Generalized Compare-and-Swap Object O

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CAS(val1, val2)
  current := O
  if current = val1 then
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```
GCAS(C, val1, val2)
  current := O
  if current C val1 then
    O := val2
    return True
  return False
```

Generalized Compare-and-Swap Object O

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For our purposes C is in {<, =, >}

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For our purposes C is in {<, =, >}, so the additional cost of implementing GCAS in hardware should be small relative to CAS...

So what does GCAS give us?

A simple **priority-based helping mechanism** for wait-freedom

Help Mechanism — Basic Idea

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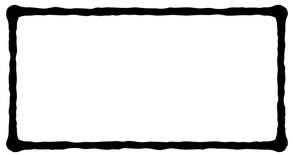

A

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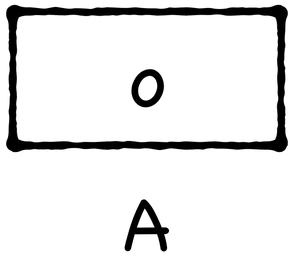

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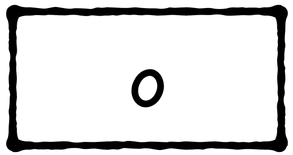
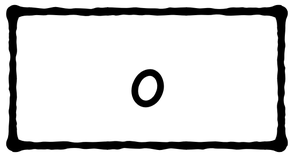


A

All processes **help** to execute the operation o currently in A

Help Mechanism — Basic Idea

To execute an operation o' process p



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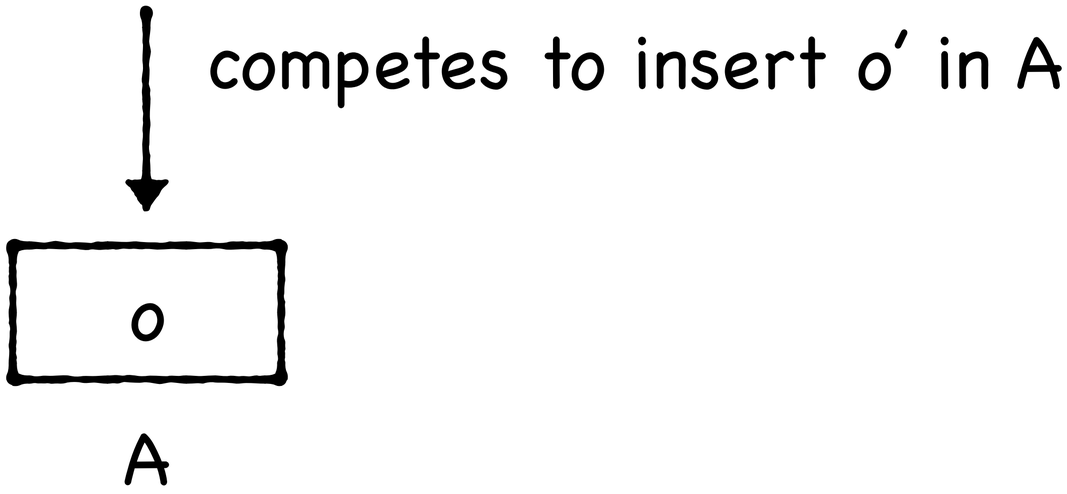

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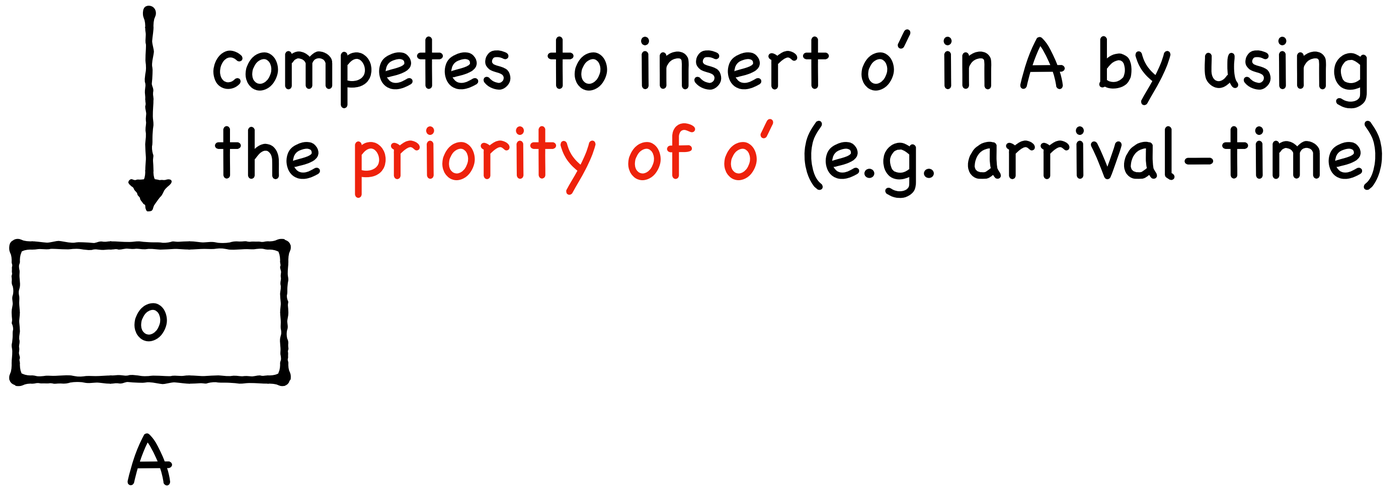

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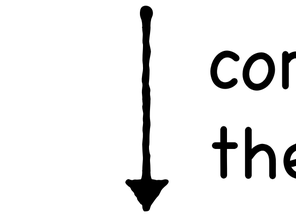
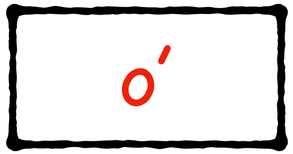


competes to insert o′ in A by using the priority of o′ (e.g. arrival-time)



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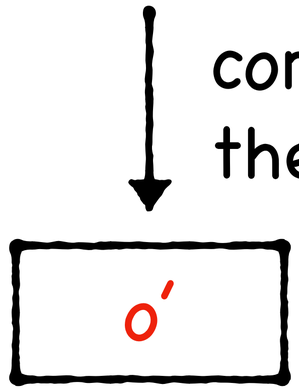

competes to insert o′ in A by using the priority of o′ (e.g. arrival-time)

All processes **help** to execute the operation o′ currently in A

Eventually the highest priority operation stays in A until it is "done"

Help Mechanism — Using GCAS to Compete

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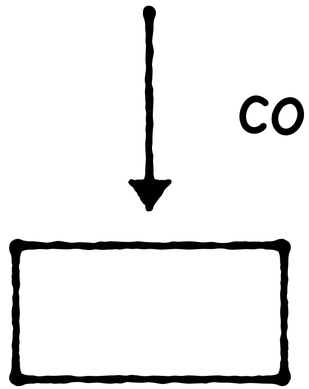
To execute an operation o′ process p

Help Mechanism — Using GCAS to Compete

To execute an operation (t', o') process p

Help Mechanism — Using GCAS to Compete

To execute an operation (t′, o′) process p



competes to place (t′, o′) into

GCAS "announcement" object A

Help Mechanism — Using GCAS to Compete

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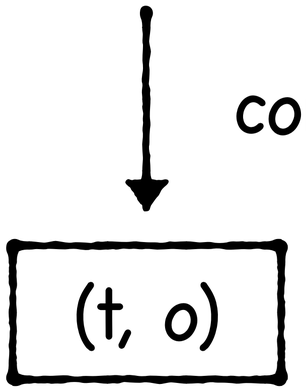

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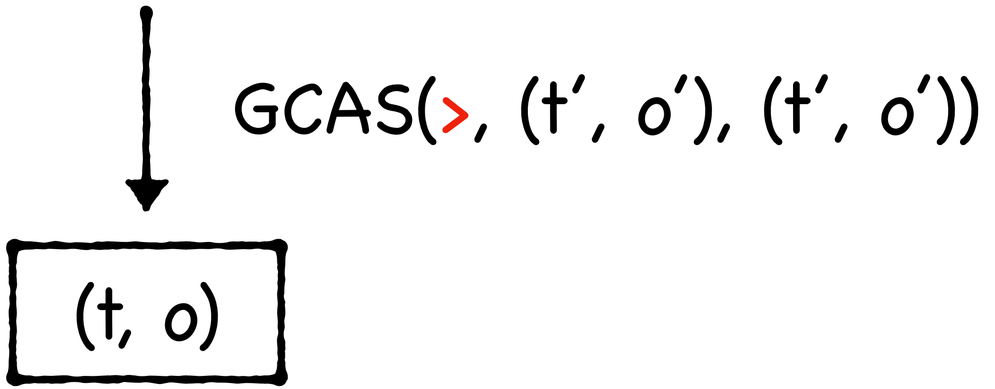

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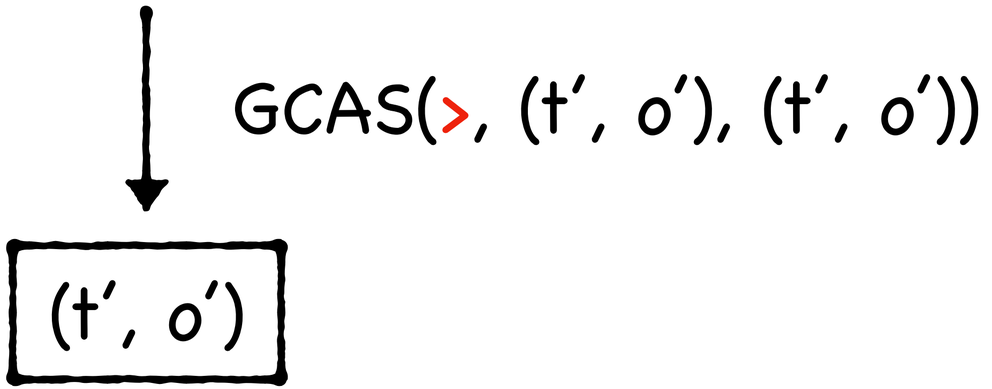

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- Eventually (t′, o′) enters A and stays there until o′ is "done"

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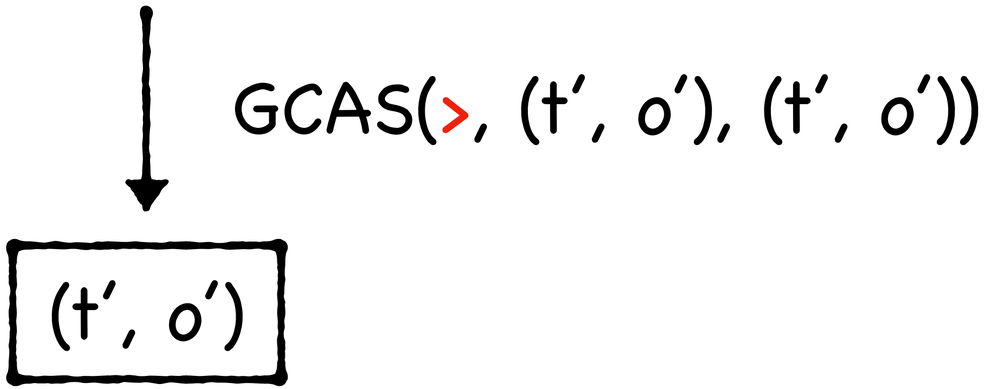
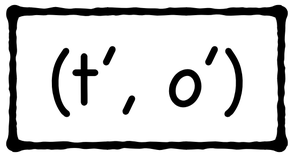


GCAS "announcement" object A

- Eventually (t′, o′) enters A and stays there until o′ is "done"
- But how do processes **remove** (t′, o′) from A after o′ is "done"?

Help Mechanism — Using GCAS to Compete

To execute an operation (t*, o*) process q



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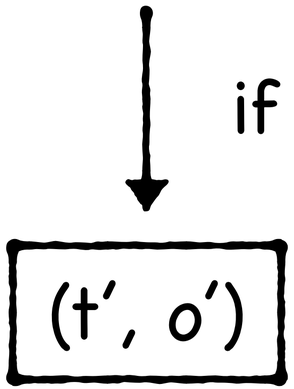

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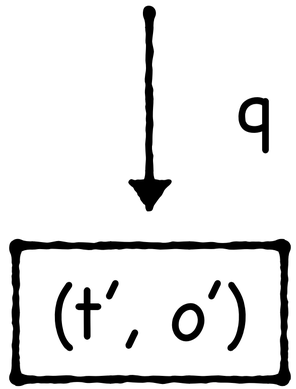

q tries to write (t*, o*) into A

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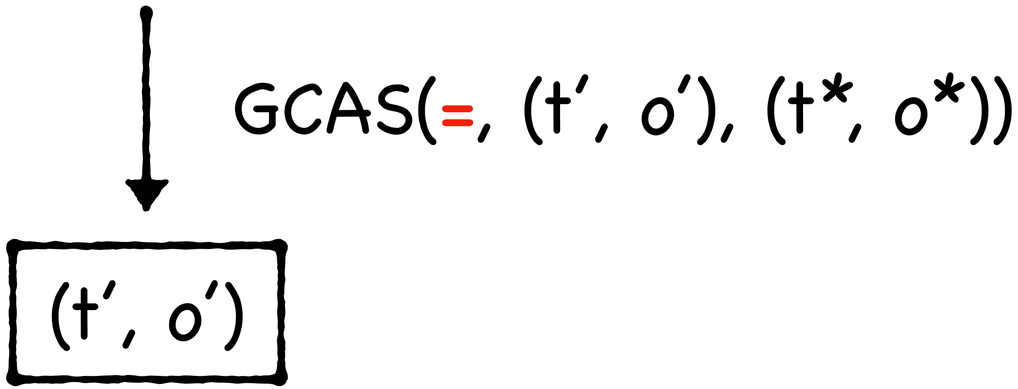

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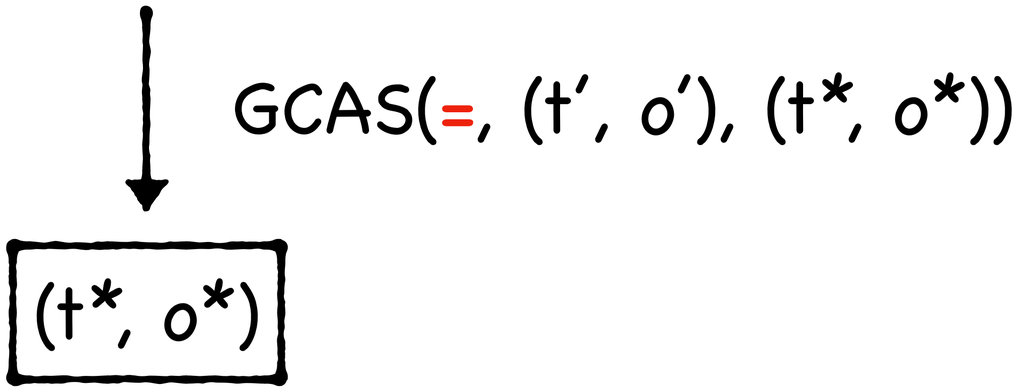

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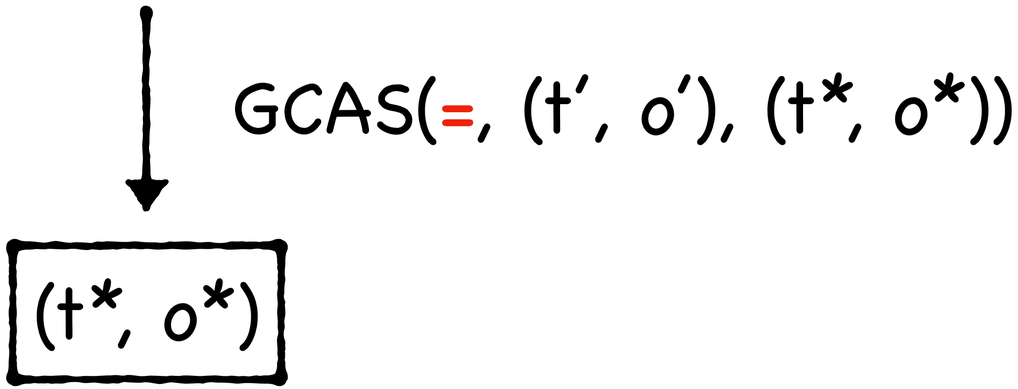

GCAS "announcement" object A

Note: we used both the > and = comparators of GCAS

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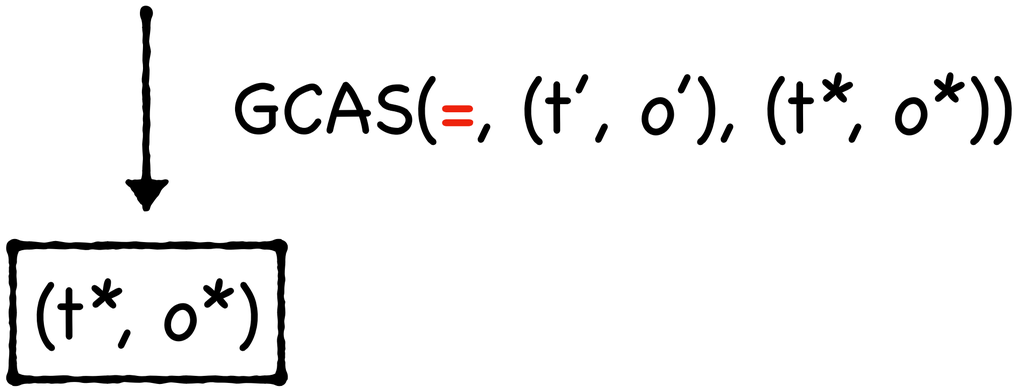

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- Comparator > : to compete on priorities

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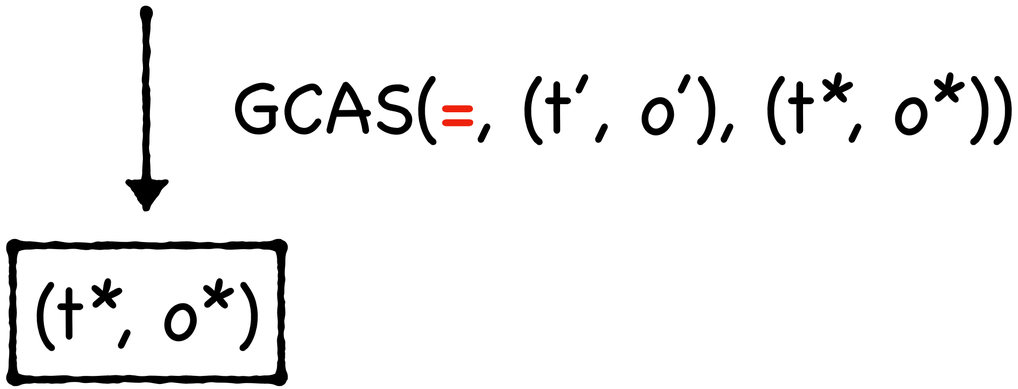

GCAS "announcement" object A

Note: we used both the > and = comparators of GCAS

- Comparator > : to compete on priorities
- Comparator = : to dislodge an operation that was "done"

Two Space-Efficient Wait-free Universal Constructions for the Infinite-Arrival Model

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There are **infinitely many processes**

Two Space-Efficient Wait-free Universal Constructions for the **Infinite-Arrival** Model

There are **infinitely many processes**

So we do **not** assume the number n of processes is known or fixed

Two Space-Efficient Wait-free Universal Constructions for the Infinite-Arrival Model

(1) Space linear in the # of processes that have participated so far

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- uses a novel memory recycling scheme

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- but assumes bounded concurrency

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Two Space-Efficient Wait-free Universal Constructions for the Infinite-Arrival Model

(1) Space linear in the # of processes that have participated so far

- **Time** linear in the point contention

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- uses a novel memory recycling scheme
- but assumes bounded concurrency

Related Work

Universal Constructions for the Infinite-Arrival Model

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[Aspnes,Shah,Shah PODC'02]: space infinite in every run

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Memory Recycling (a lot of work on this topic!)

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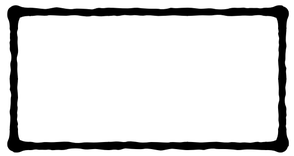
[Detlefs,Martin,Moir,Steele'01], [Herlihy,Luchangco,Martin,Moir'05], [Nikolaev,Ravindran'21]: work for this model but are not wait-free

Our first universal construction

Base Objects

Base Objects

C  a Fetch&Inc to timestamp operations

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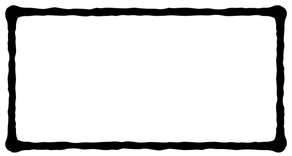
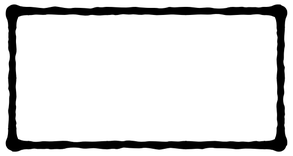
C  a Fetch&Inc to timestamp operations

R_p  a CAS to store the response of the last operation of p

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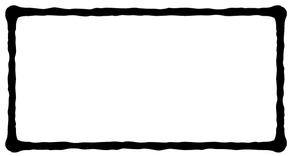
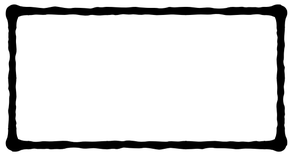
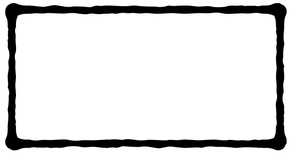
C  a Fetch&Inc to timestamp operations

R_p  a CAS to store the response of the last operation of p

A  a GCAS "announcement" object

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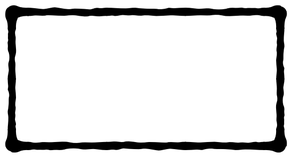
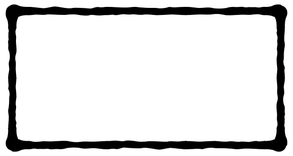
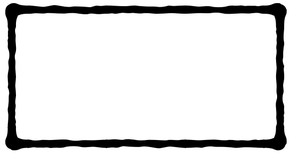
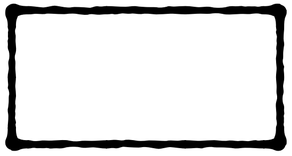
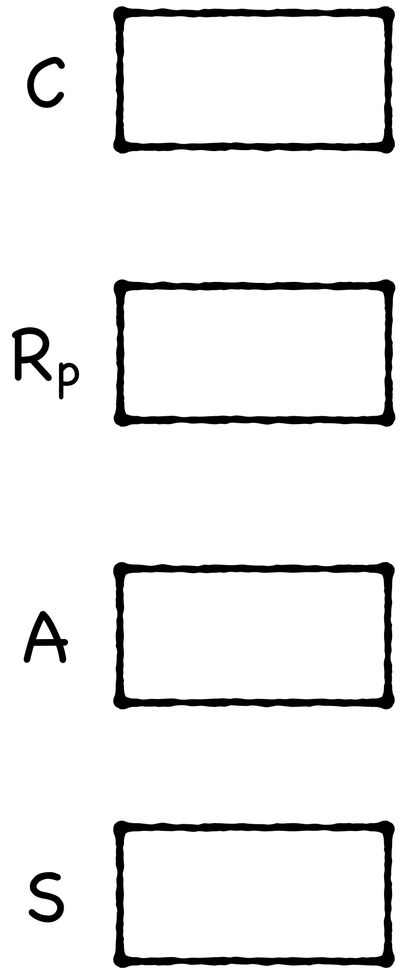
C  a Fetch&Inc to timestamp operations

R_p  a CAS to store the response of the last operation of p

A  a GCAS "announcement" object

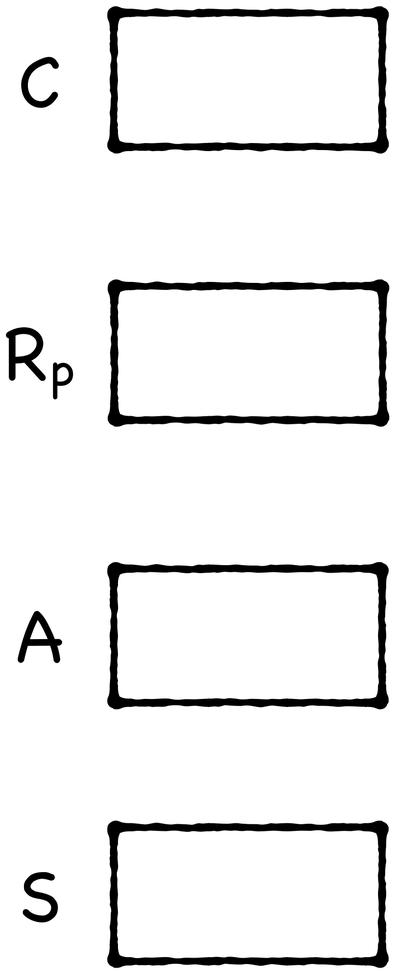
S  a CAS to store the state of the implemented object

Base Objects




```
DoOp(o) for process p
  if o is the first op of p then Rp := get object from manager
  t := C.FI()
  Rp := (t, NIL)
  while Rp = (t, NIL) do
    (t*, s*, r*, R*) := S
    R*.CAS((t*, NIL), (t*, r*))
    A.GCAS(>, (t, o, Rp), (t, o, Rp))
    (t', o', R') := A
    if R' = (t', NIL) then
      (s', r') := apply(o', s*)
      S.CAS((t*, s*, r*, R*), (t', s', r', R'))
    else
      A.GCAS(=, (t', o', R'), (t, o, Rp))
  r := Rp.response
  return r
```

Base Objects

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    (t*, s*, r*, R*) := S
    R*.CAS((t*, NIL), (t*, r*))
    A.GCAS(>, (t, o, Rp), (t, o, Rp))
    (t', o', R') := A
    if R' = (t', NIL) then
      (s', r') := apply(o', s*)
      S.CAS((t*, s*, r*, R*), (t', s', r', R'))
    else
      A.GCAS(=, (t', o', R'), (t, o, Rp))
  r := Rp.response
  return r
```

Base Objects

C

R_p

A

S

DoOp(o) for process p

if o is the **first** op of p then R_p := get object from manager

t := C.FI()

R_p := (t, NIL)

while R_p =

(t*, s*

R*.CA

A.GC

(t', o

if R'

(s', r

S.CAS(

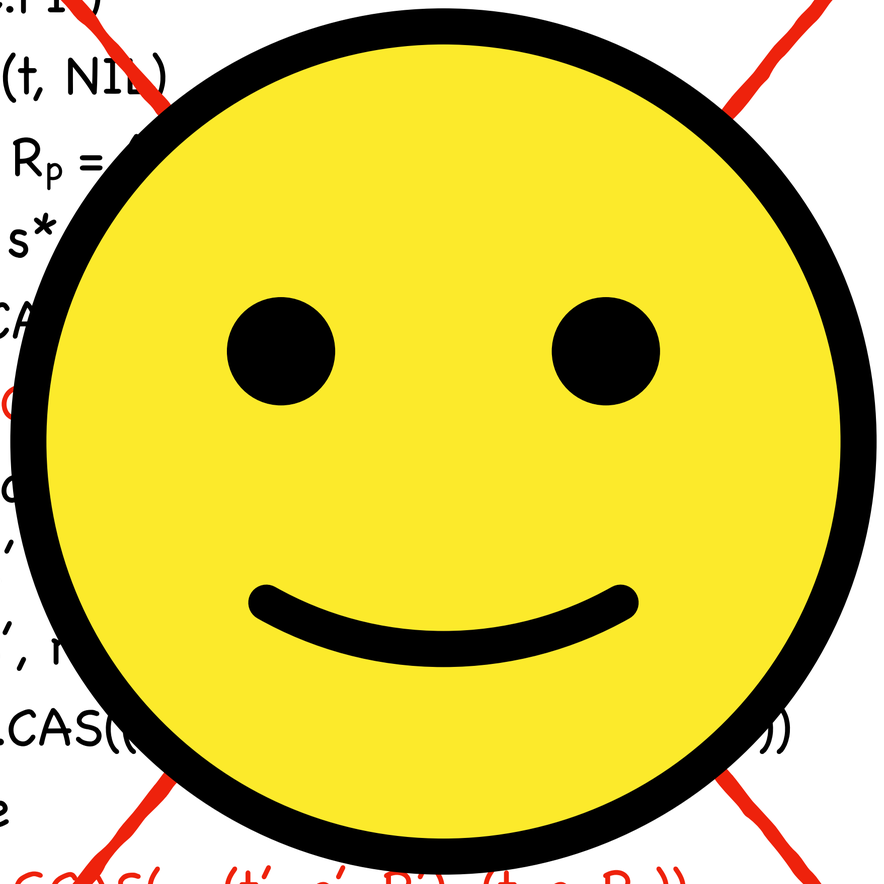
))

else

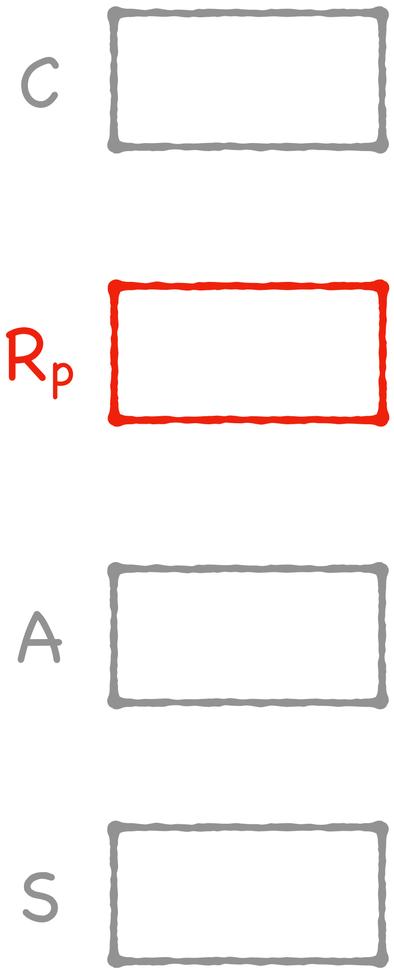
A.GCAS(=, (t', o', R'), (t, o, R_p))

r := R_p.response

return r

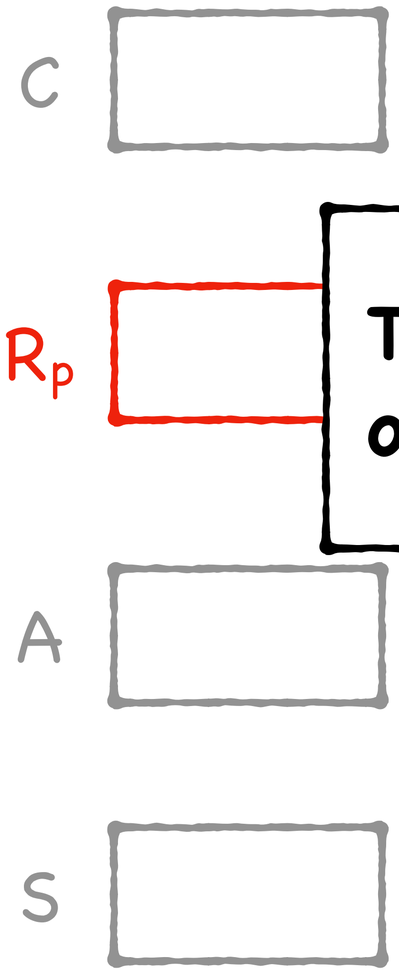


Base Objects




```
DoOp(o) for process p
  if o is the first op of p then Rp := get object from manager
  t := C.FI()
  Rp := (t, NIL)
  while Rp = (t, NIL) do
    (t*, s*, r*, R*) := S
    R*.CAS((t*, NIL), (t*, r*))
    A.GCAS(>, (t, o, Rp), (t, o, Rp))
    (t', o', R') := A
    if R' = (t', NIL) then
      (s', r') := apply(o', s*)
      S.CAS((t*, s*, r*, R*), (t', s', r', R'))
    else
      A.GCAS(=, (t', o', R'), (t, o, Rp))
  r := Rp.response
  return r
```

Base Objects




```
DoOp(o) for process p
  if o is the first op of p then R_p := get object from manager
  t := C.FI()
  R_p := (t, NIL)
  while R_p = (t, NIL) do
    (t*, s*, r*, R*) := S
    (s', r') := apply(o', s*)
    S.CAS((t*, s*, r*, R*), (t', s', r', R'))
  else
    A.GCAS(=, (t', o', R'), (t, o, R_p))
  r := R_p.response
  return r
```

The space complexity is linear in the number of processes that have participated so far

Memory recycling in our second universal construction

Memory Manager

Memory Manager

Processes can:

Memory Manager

Processes can:

(1) get "cells" from the memory manager, and

Memory Manager

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(1) get "cells" from the memory manager, and

(2) free "cells" by returning them to the memory manager

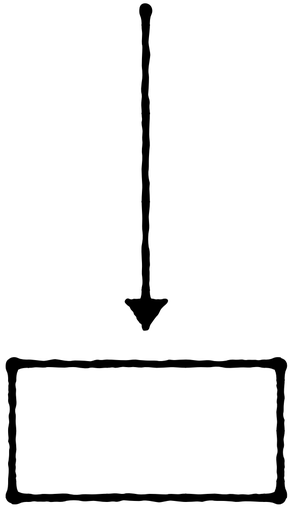
Warning:

(1) the memory manager may give any freed cell to another application, and this application may write "junk" into it, so

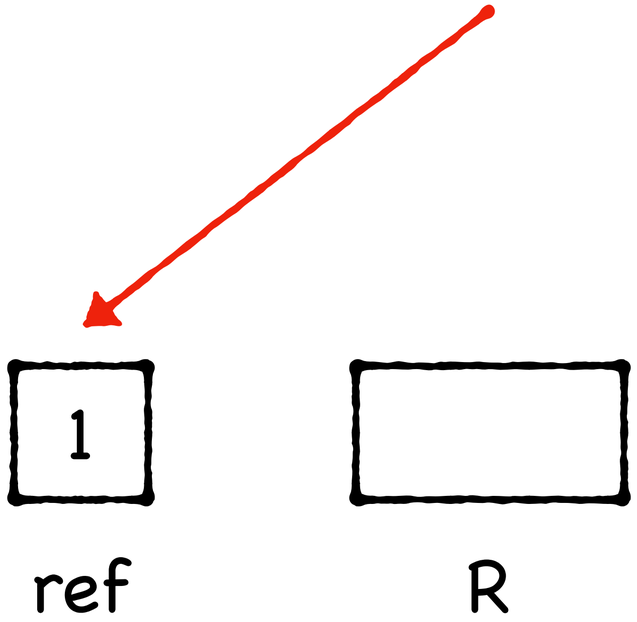
(2) processes should never access a cell that has been freed (as otherwise it may return a wrong value)

Reference Counters

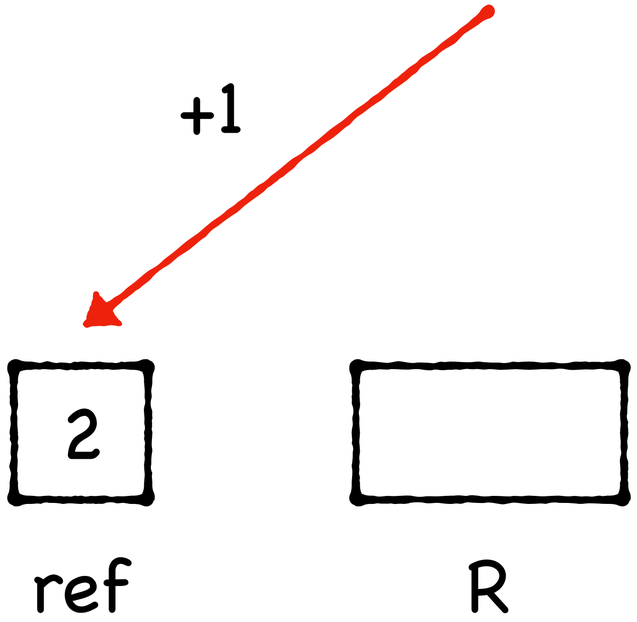
Before process p accesses cell R it must gain the "right" to access R



p does so by incrementing the reference counter for R

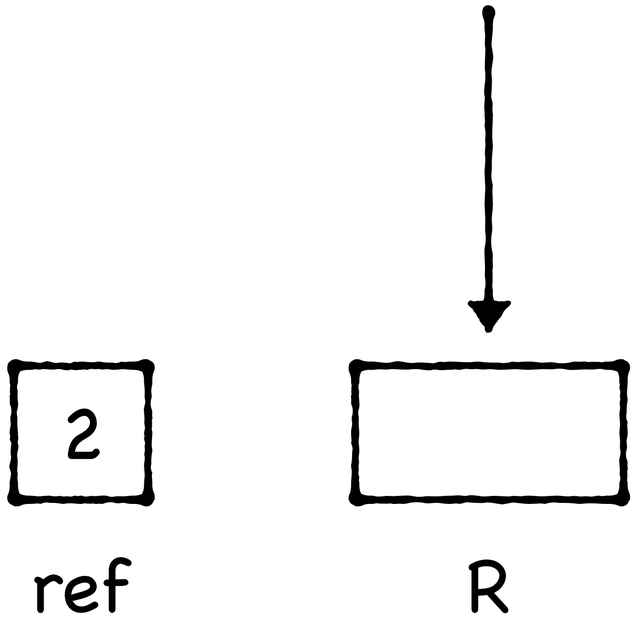


p does so by incrementing the reference counter for R

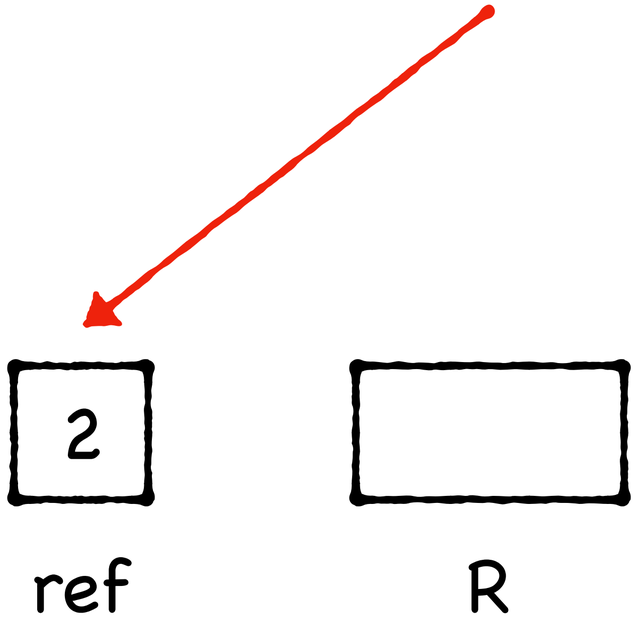


Now p has the "right" to access R

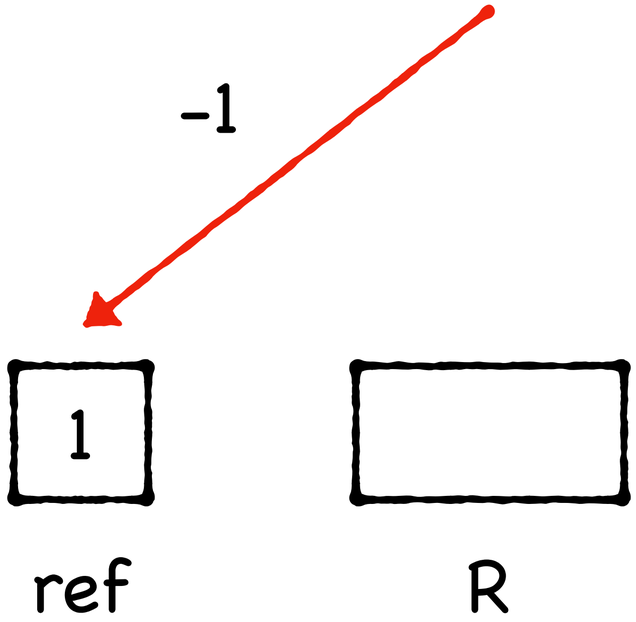
After p is done accessing cell R it relinquishes its "right" to access R



p does so by decrementing the reference counter for R

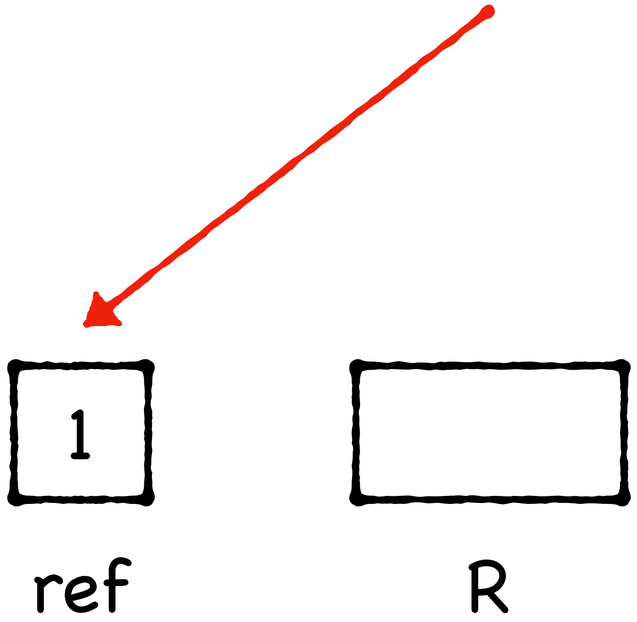


p does so by decrementing the reference counter for R

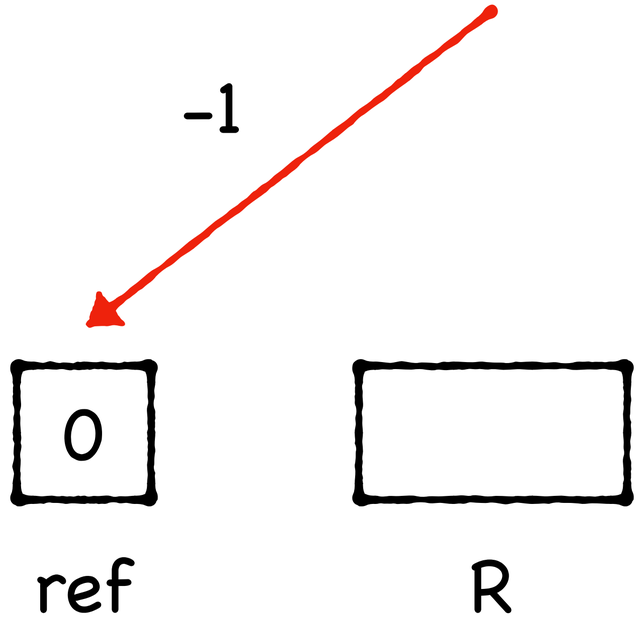


Now p relinquished access to R

To relinquishes its access to R, q decrements the reference counter for R

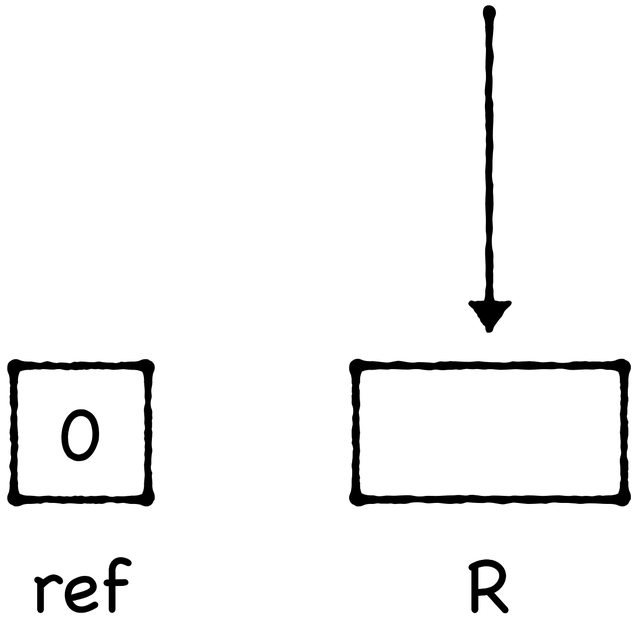


To relinquishes its access to R, q decrements the reference counter for R

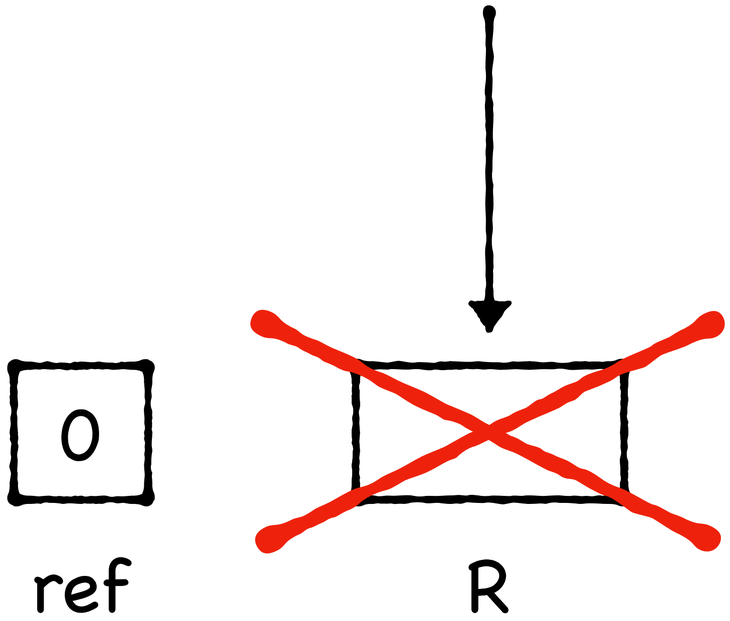


Now q relinquished access to R

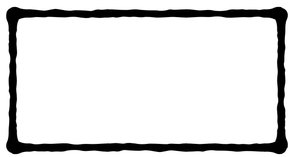
Now no process has the right to access R



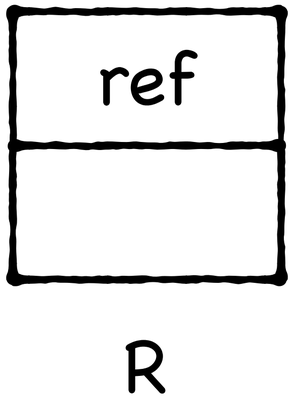
Now no process has the right to access R, so q can safely recycle it



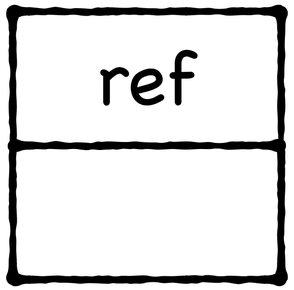
But where do we put the reference counter for R?



R

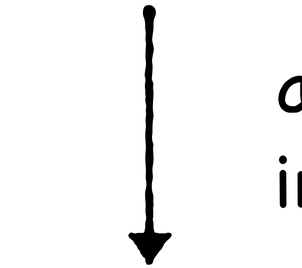

ref
R

To acquire “the right” to access R

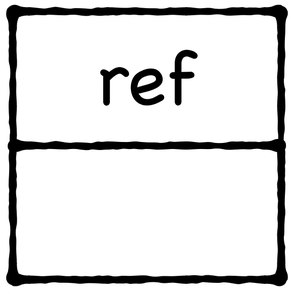


R

To acquire "the right" to access R



a process must access R to increment its reference counter!



R

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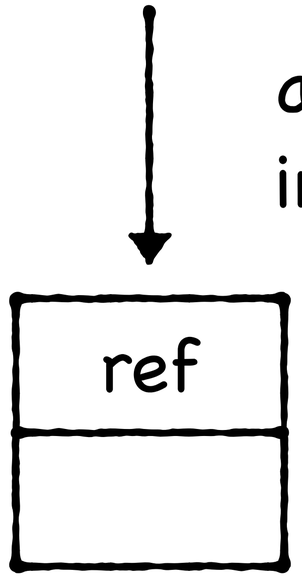
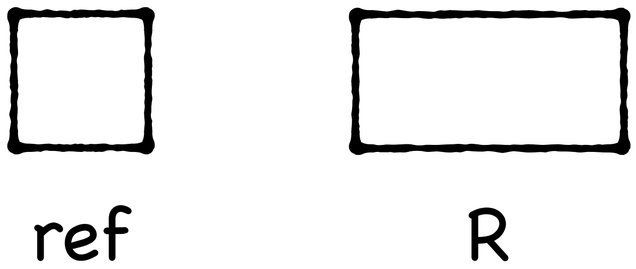


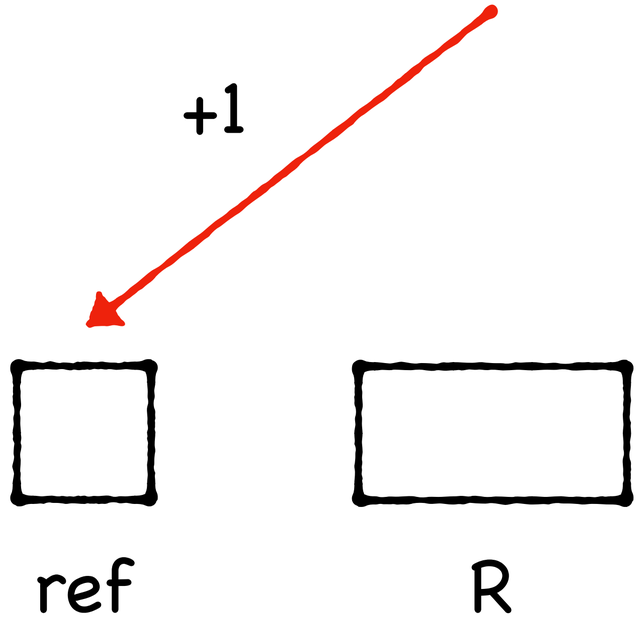
a process must access R to increment its reference counter!

R

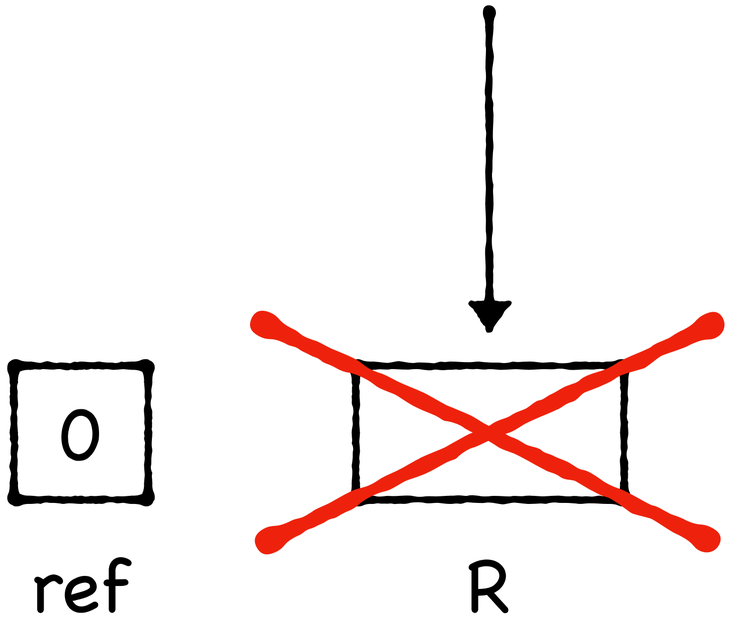
A chicken-and-egg situation


ref
R

To acquire "the right" to access R we no longer need to access R!

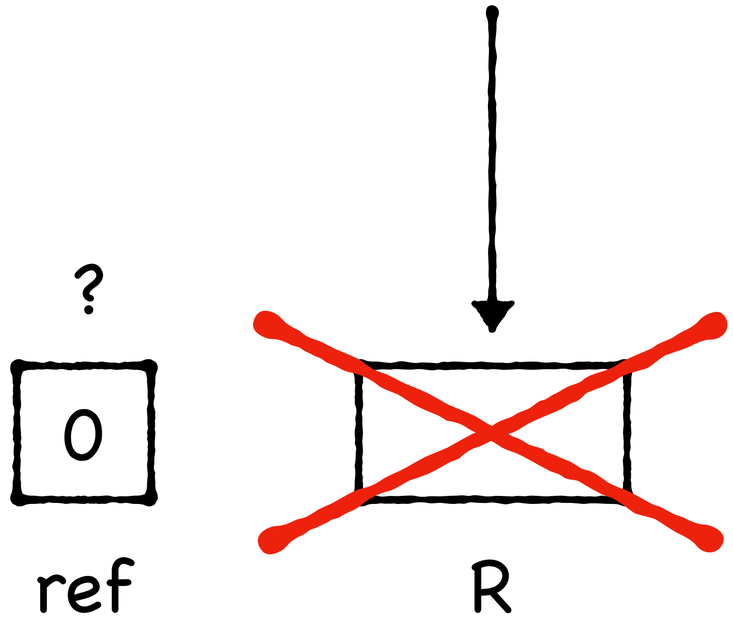


We can safely recycle a cell when its reference counter is zero



We can safely recycle a cell when its reference counter is zero

The problem: when can we safely recycle its reference counter?



We can safely recycle a cell when its reference counter is zero

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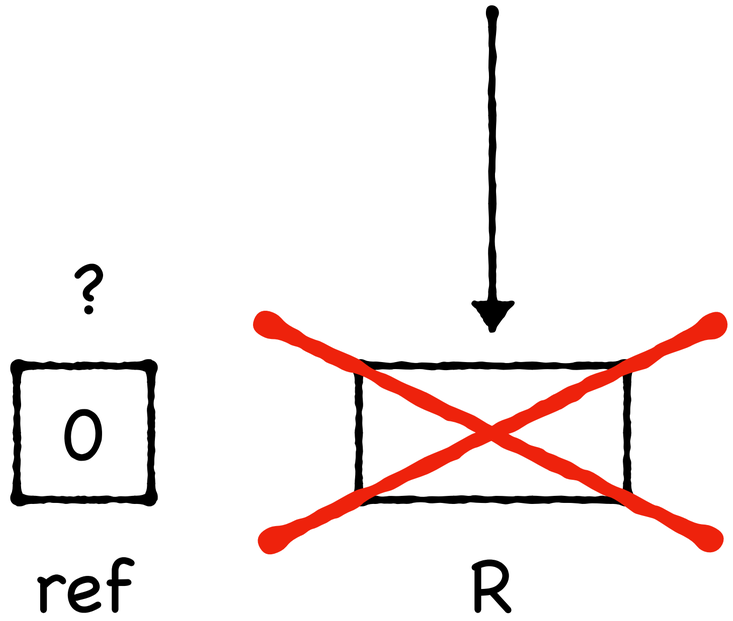
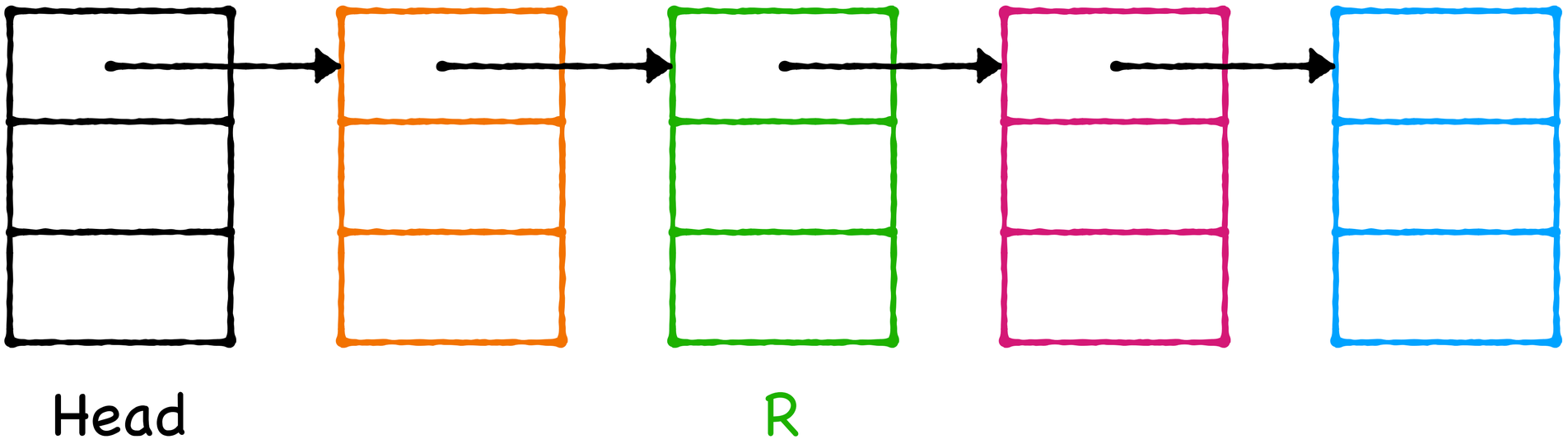


A different kind of chicken-and-egg situation!

Solving the chicken-and-egg problems

- Thread the response cells of pending operations into a list



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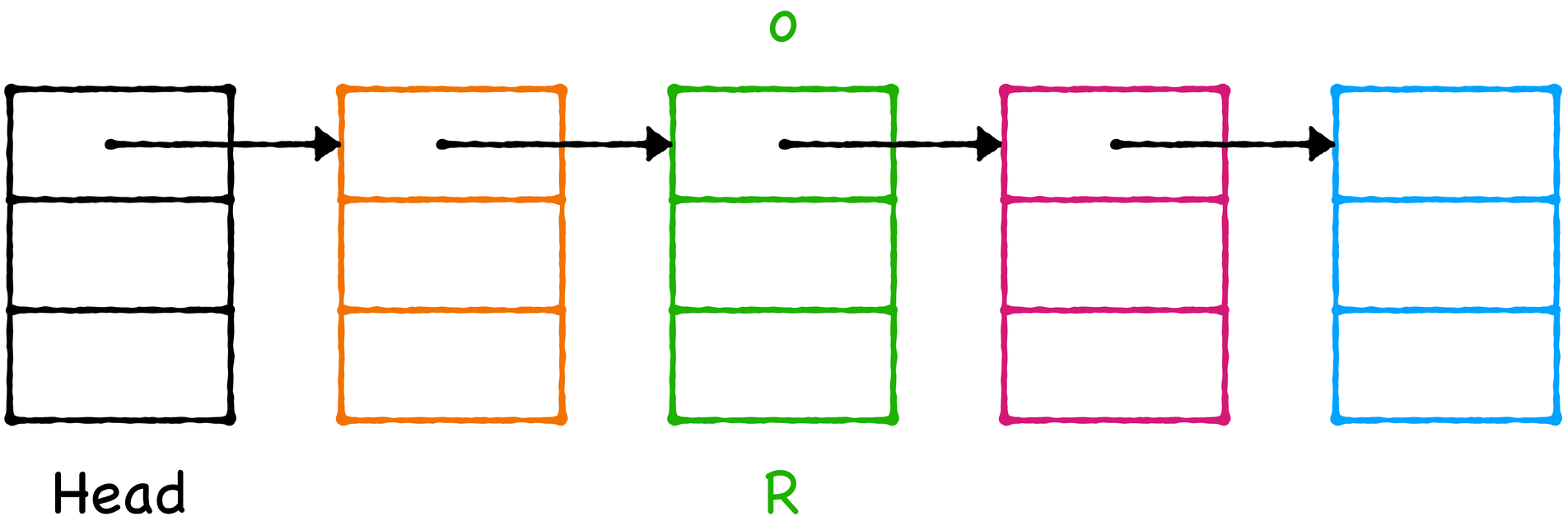
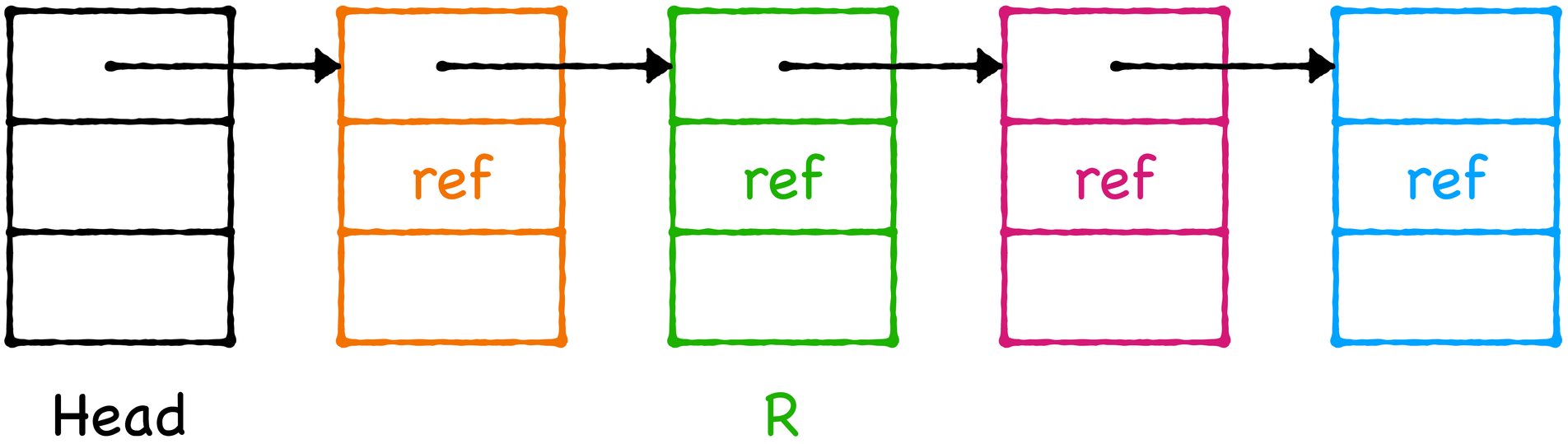
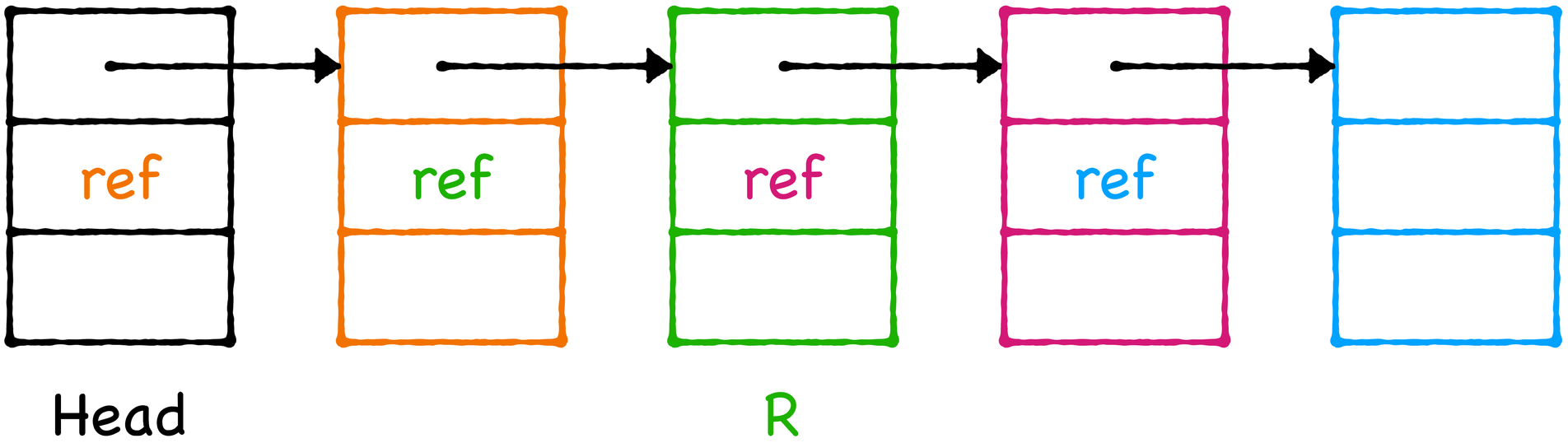


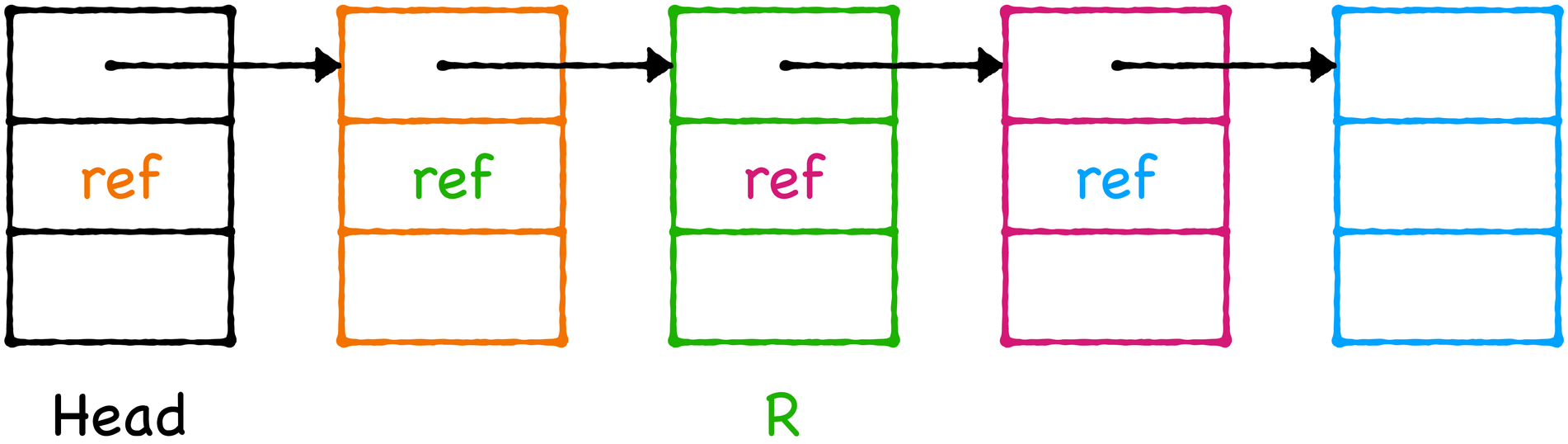
- Thread the response cells of pending operations into a list
- Put the reference counter of each cell into the cell itself



- Thread the response cells of pending operations into a list
- Put the reference counter of each cell into the cell itself



- Thread the response cells of pending operations into a list
- Put the reference counter of each cell into ~~the cell itself~~ its predecessor!



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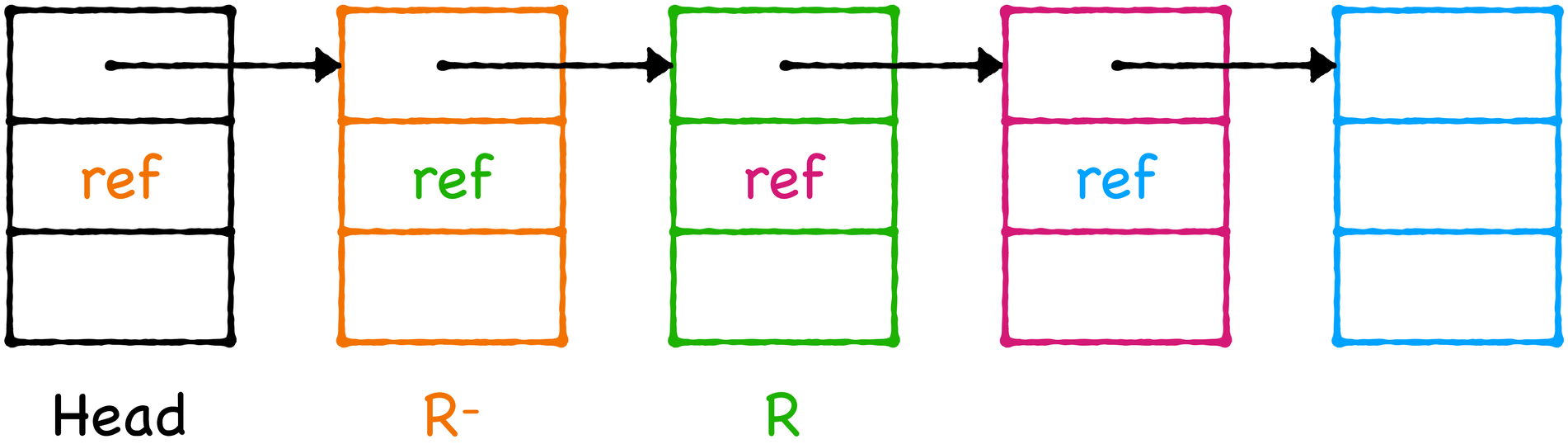

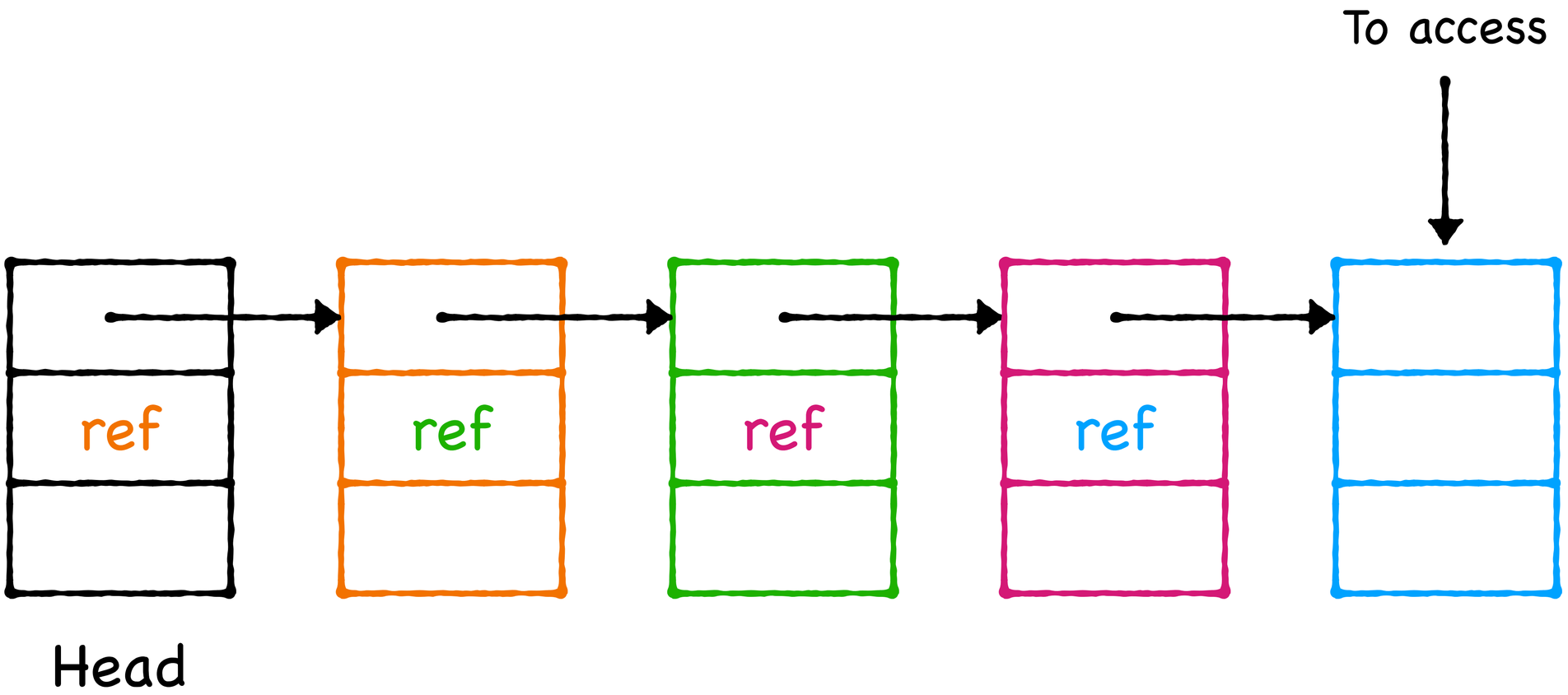
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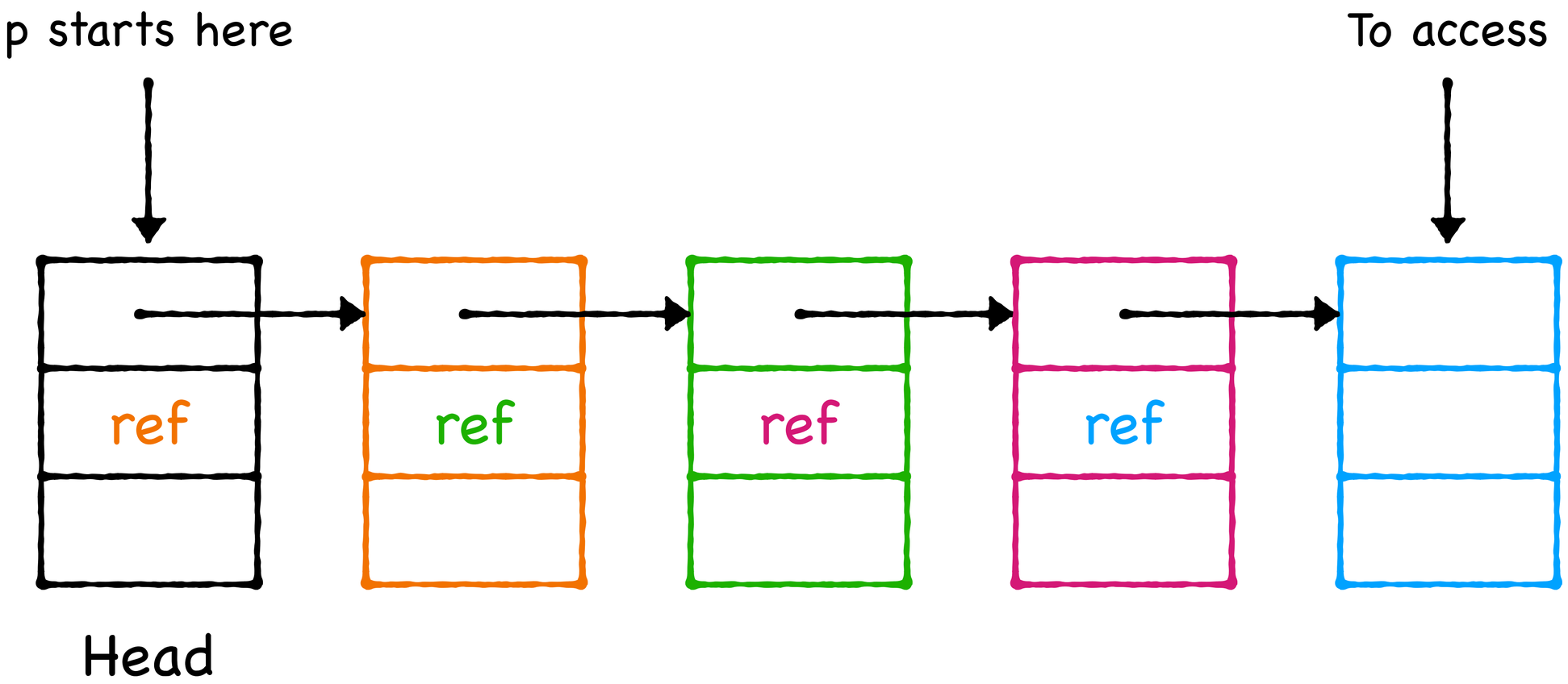
p starts here
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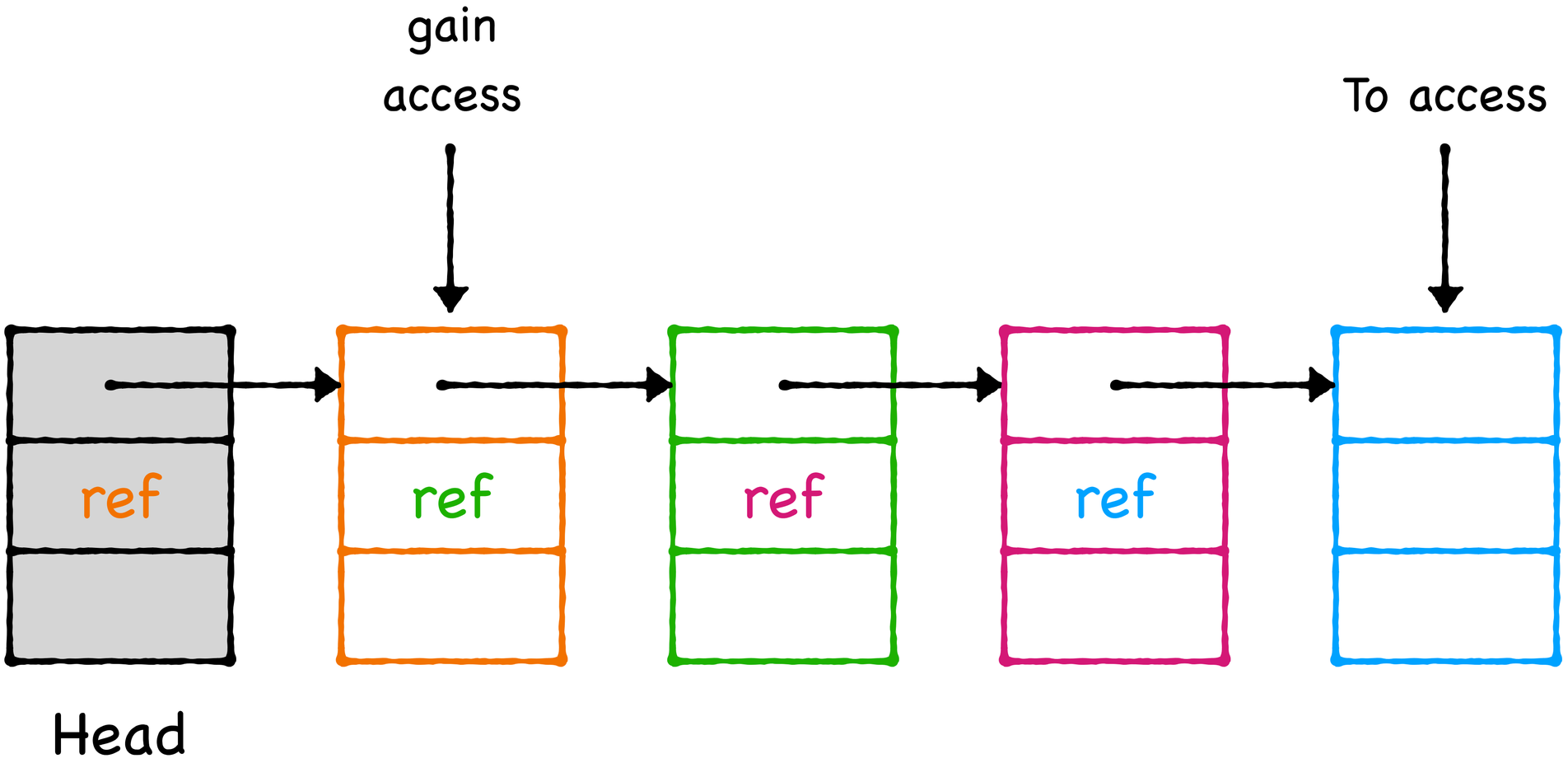
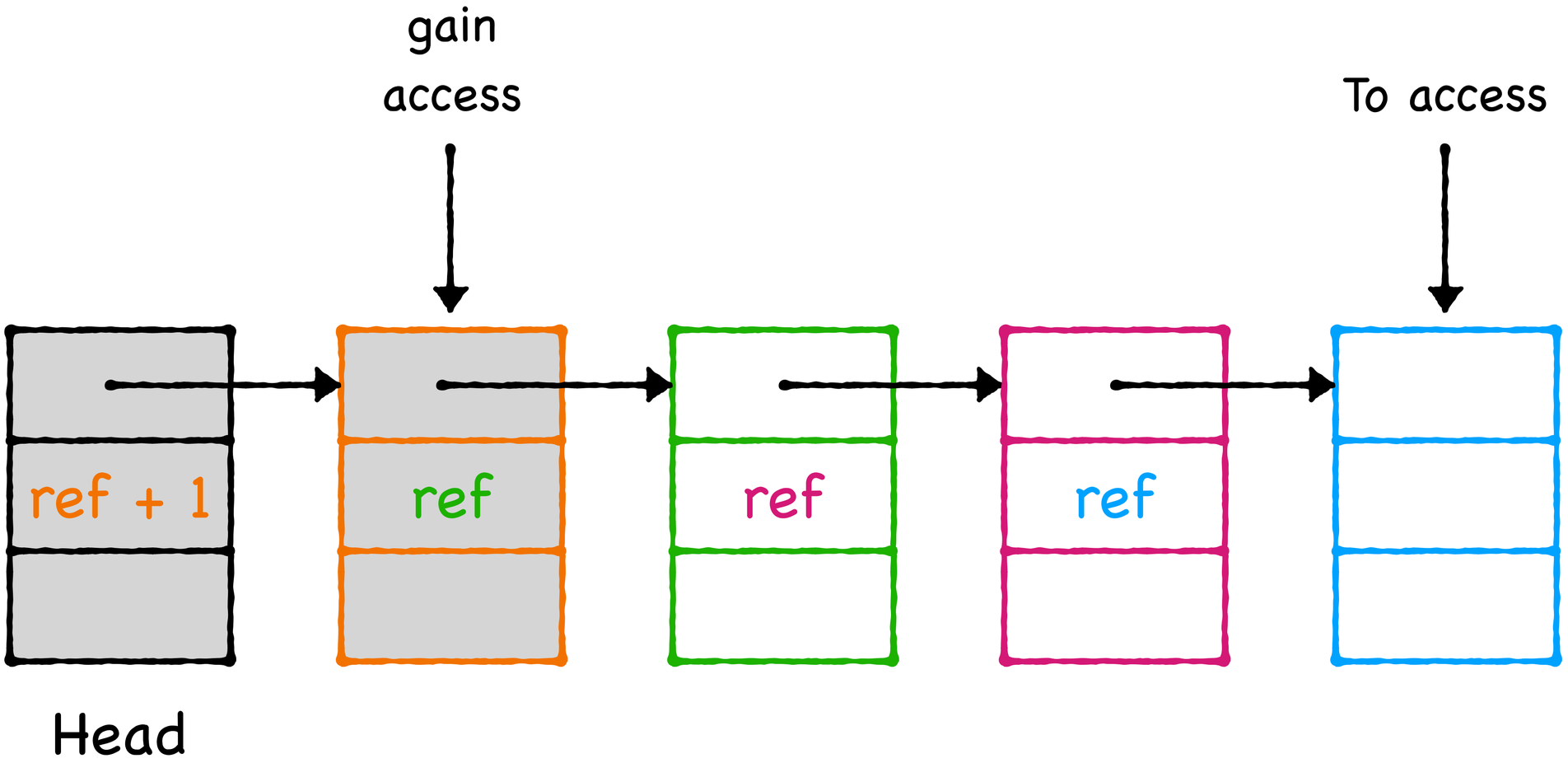
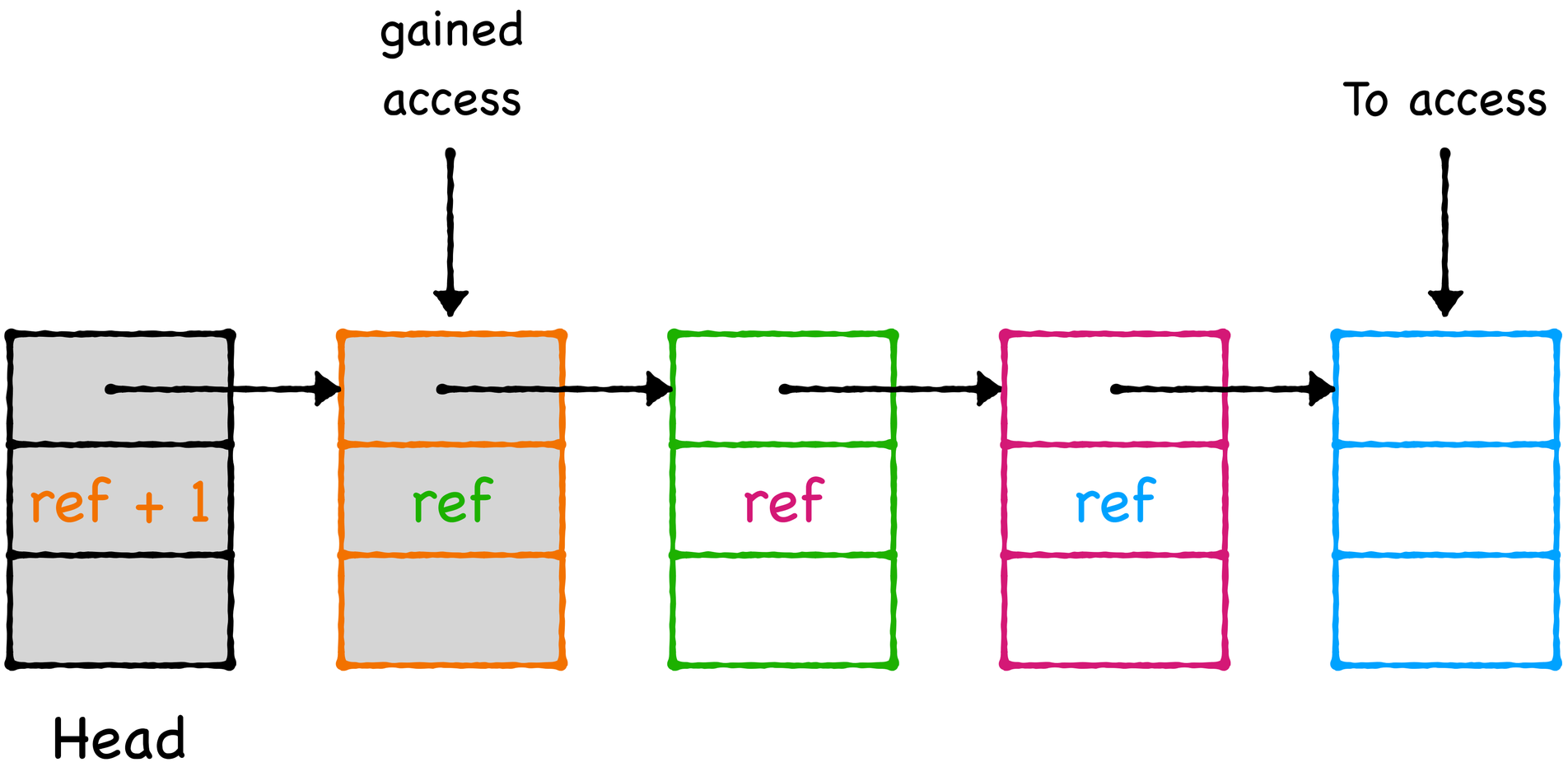


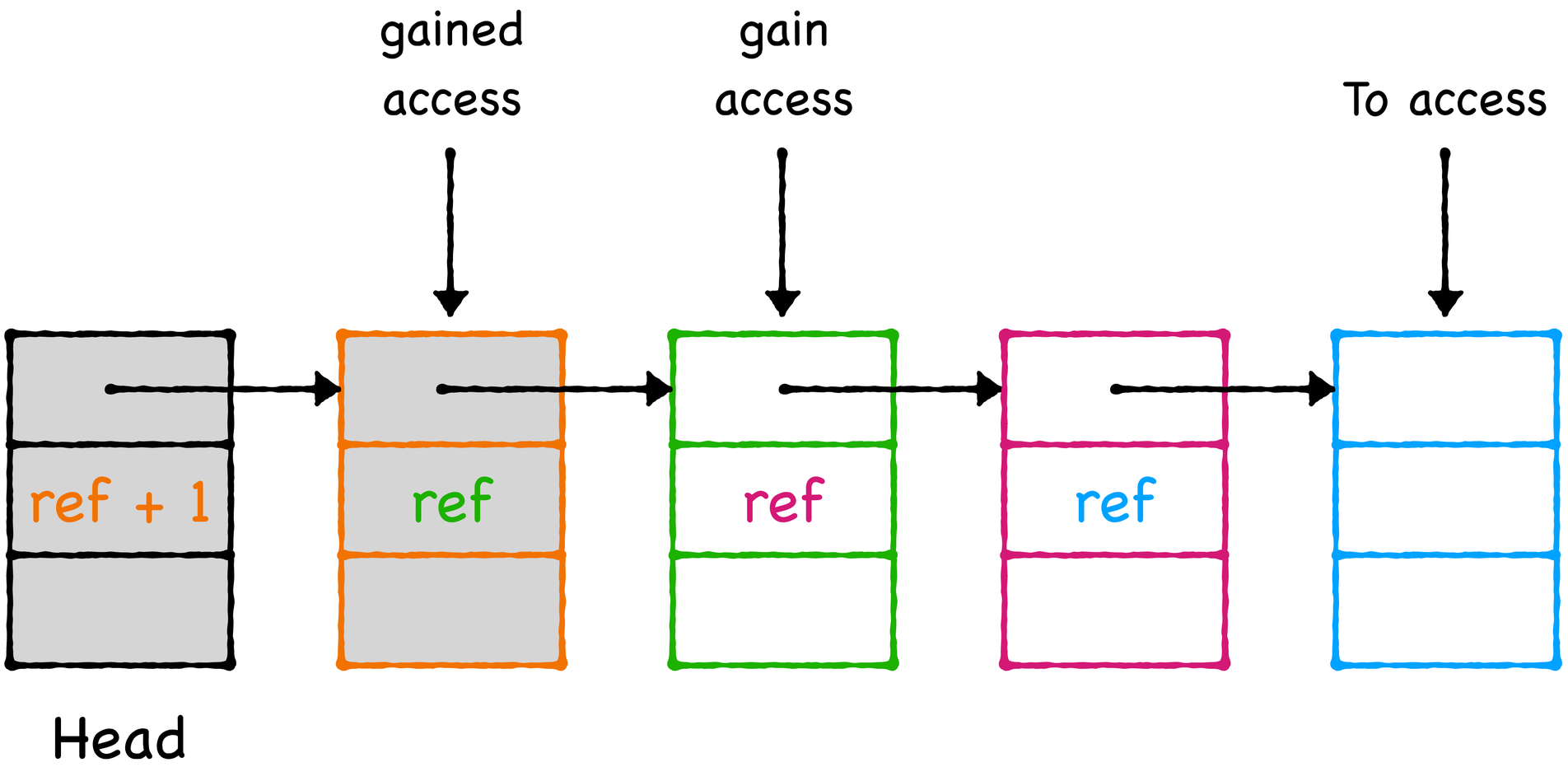
# of acquired cells: 1



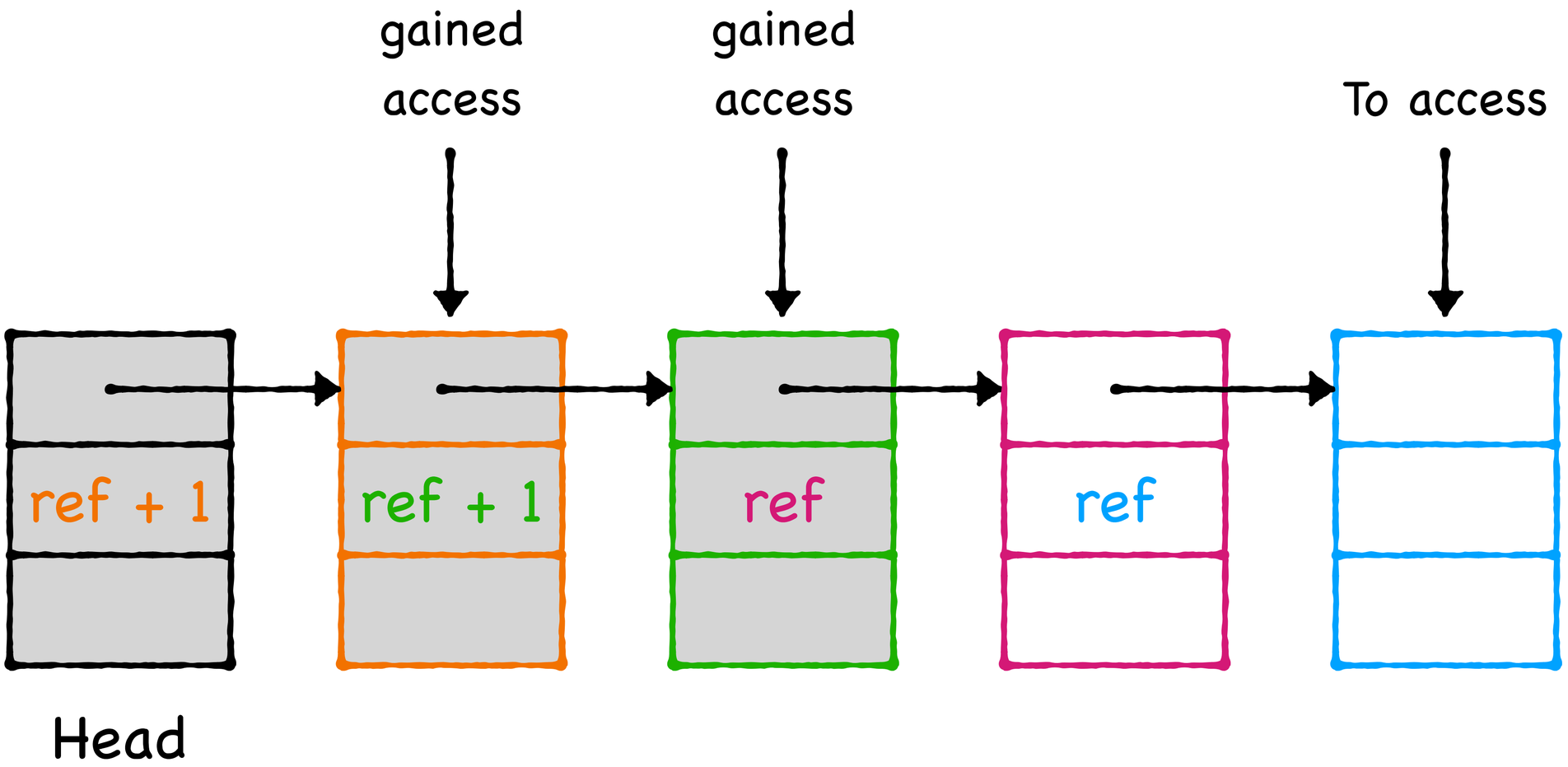
# of acquired cells: 2



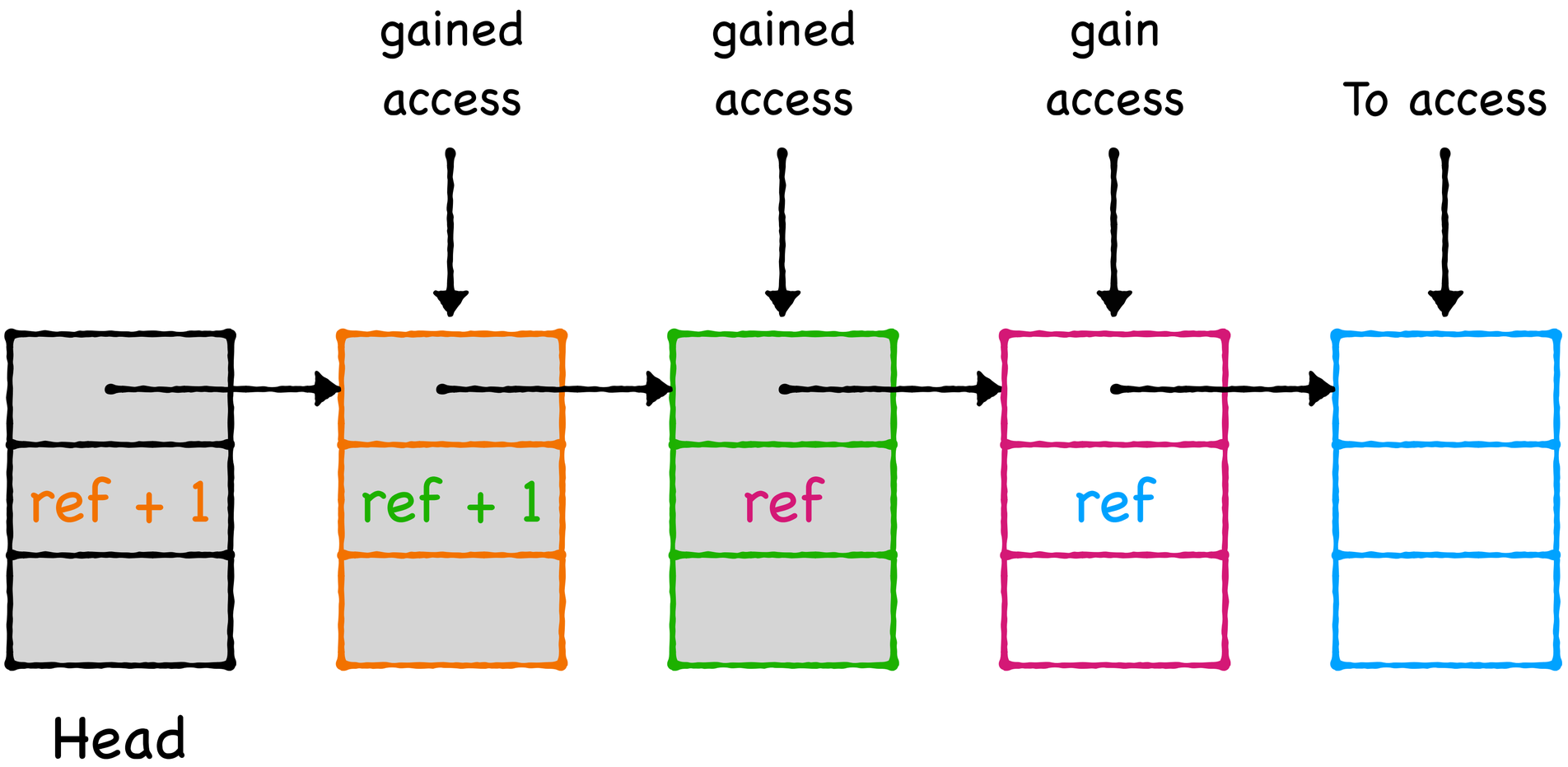
# of acquired cells: 2



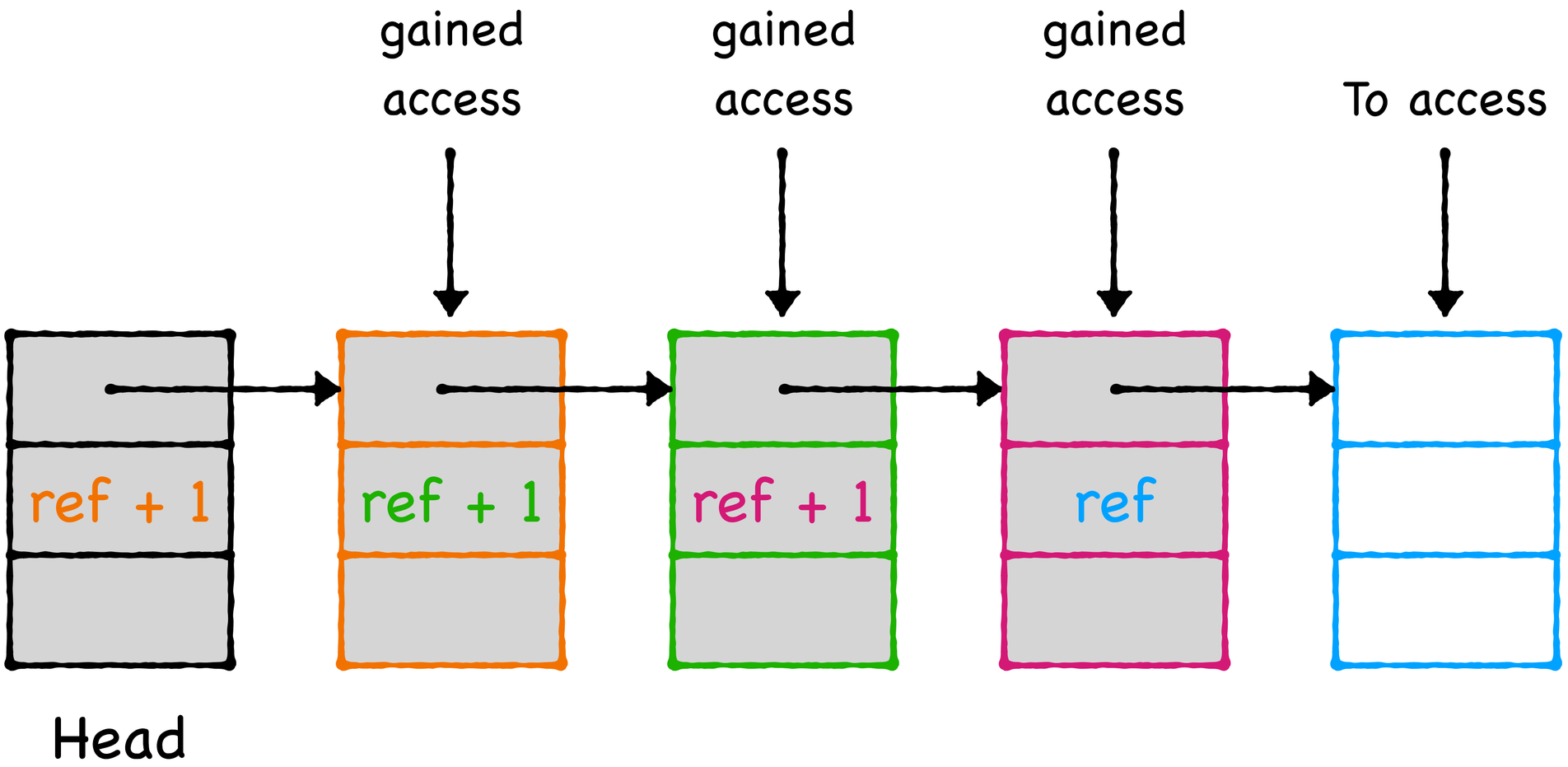
# of acquired cells: 2



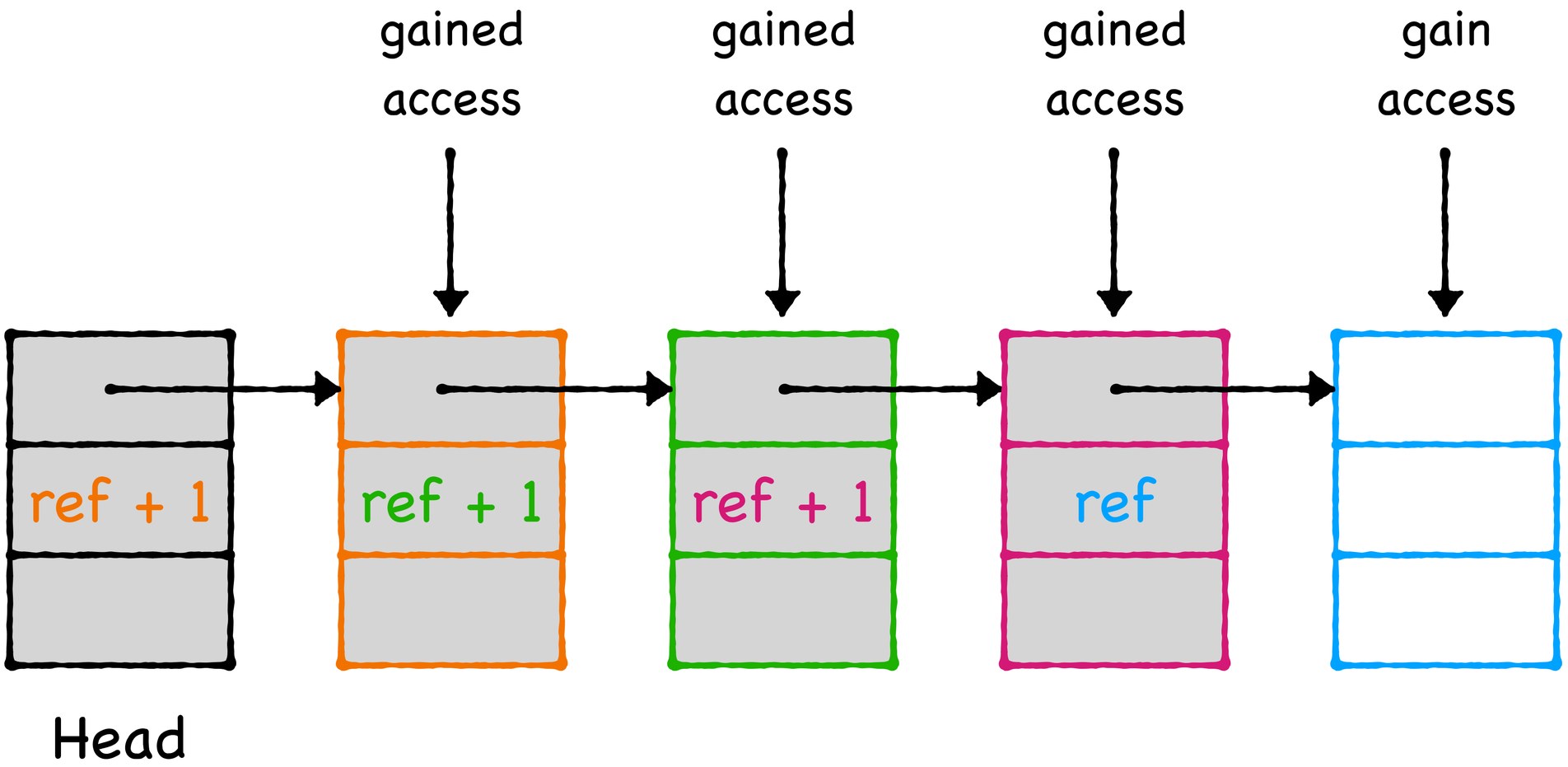
# of acquired cells: 3



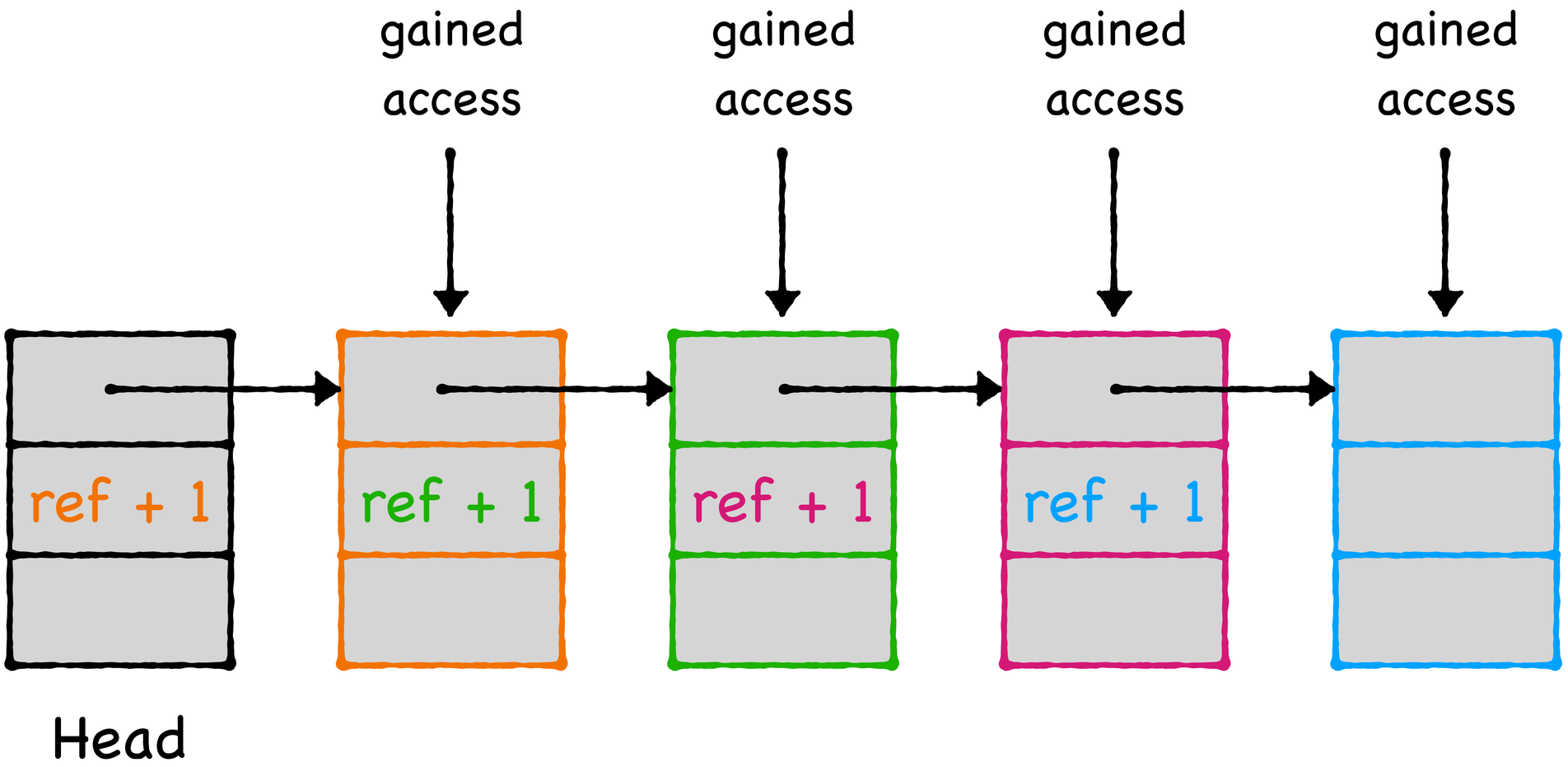
# of acquired cells: 3



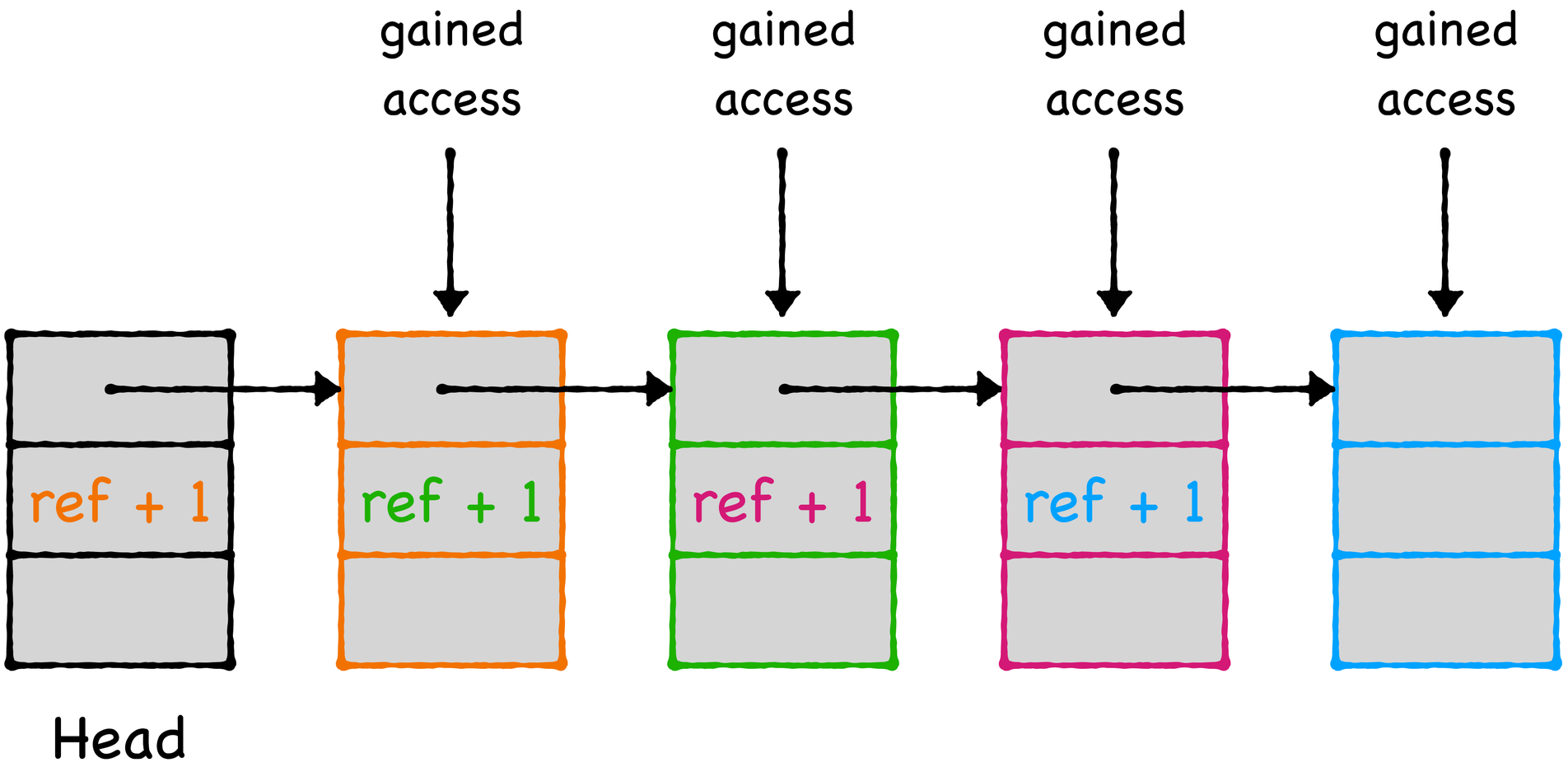
# of acquired cells: 4



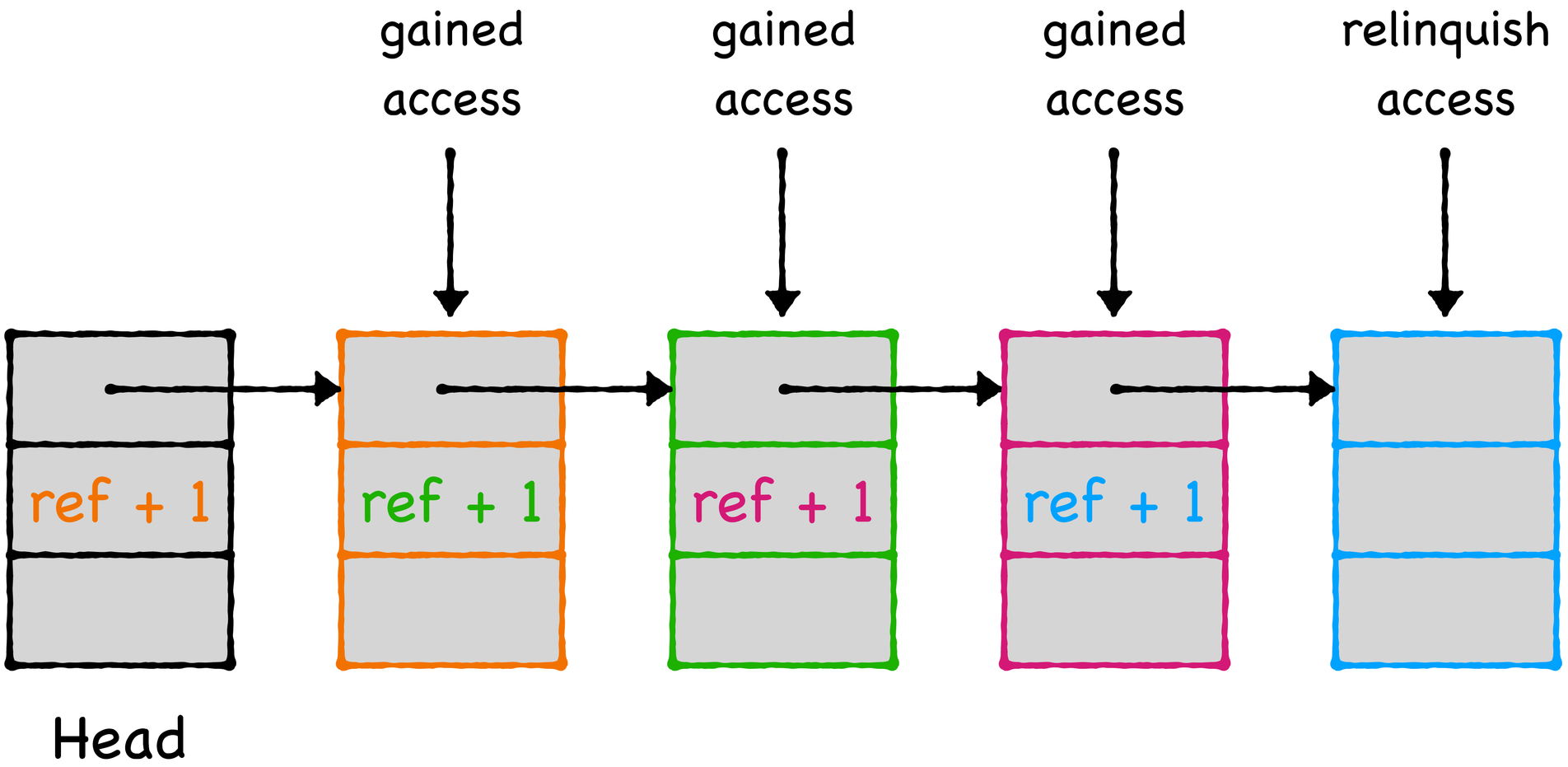
# of acquired cells: 4



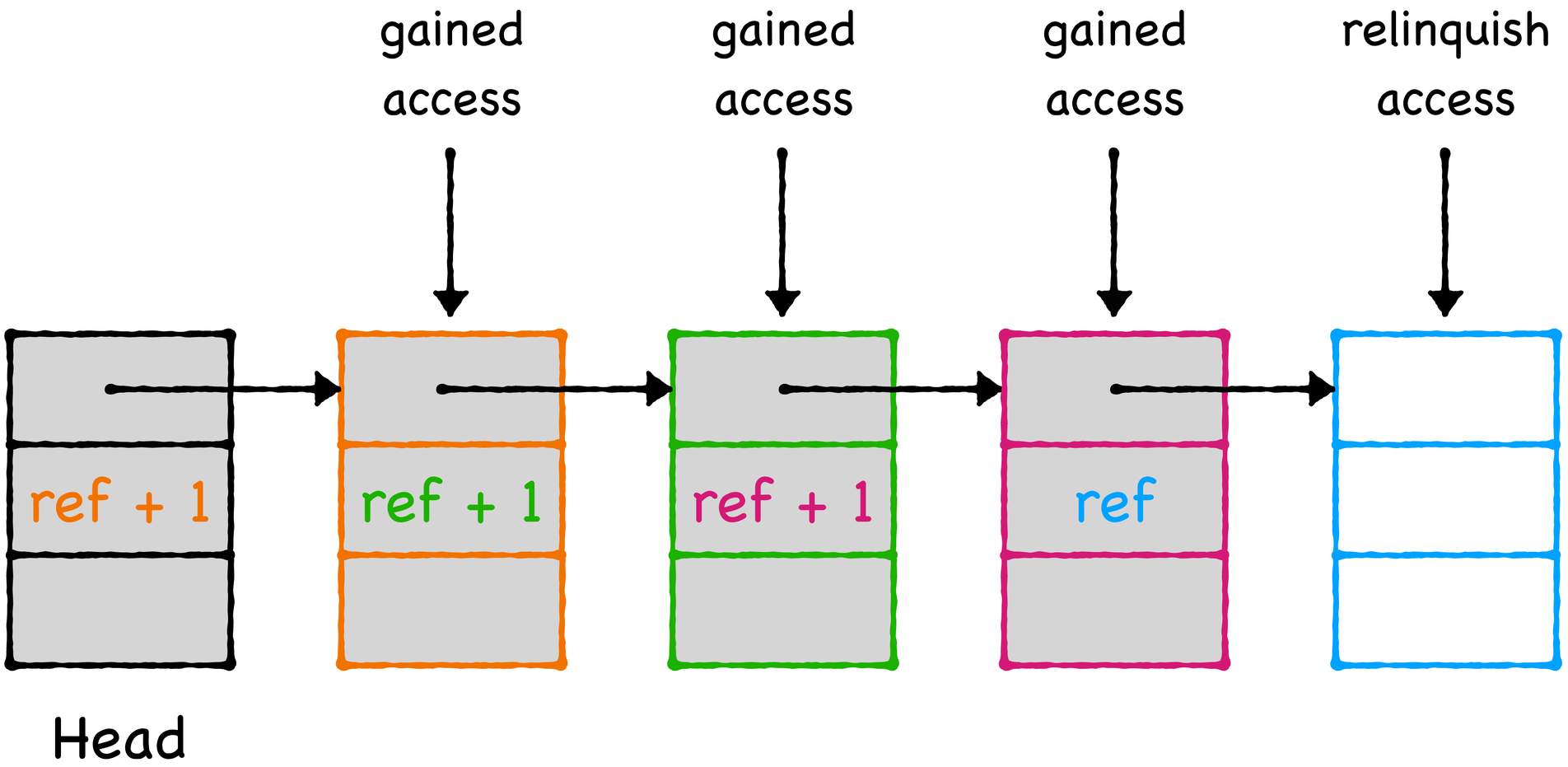
# of acquired cells: 5



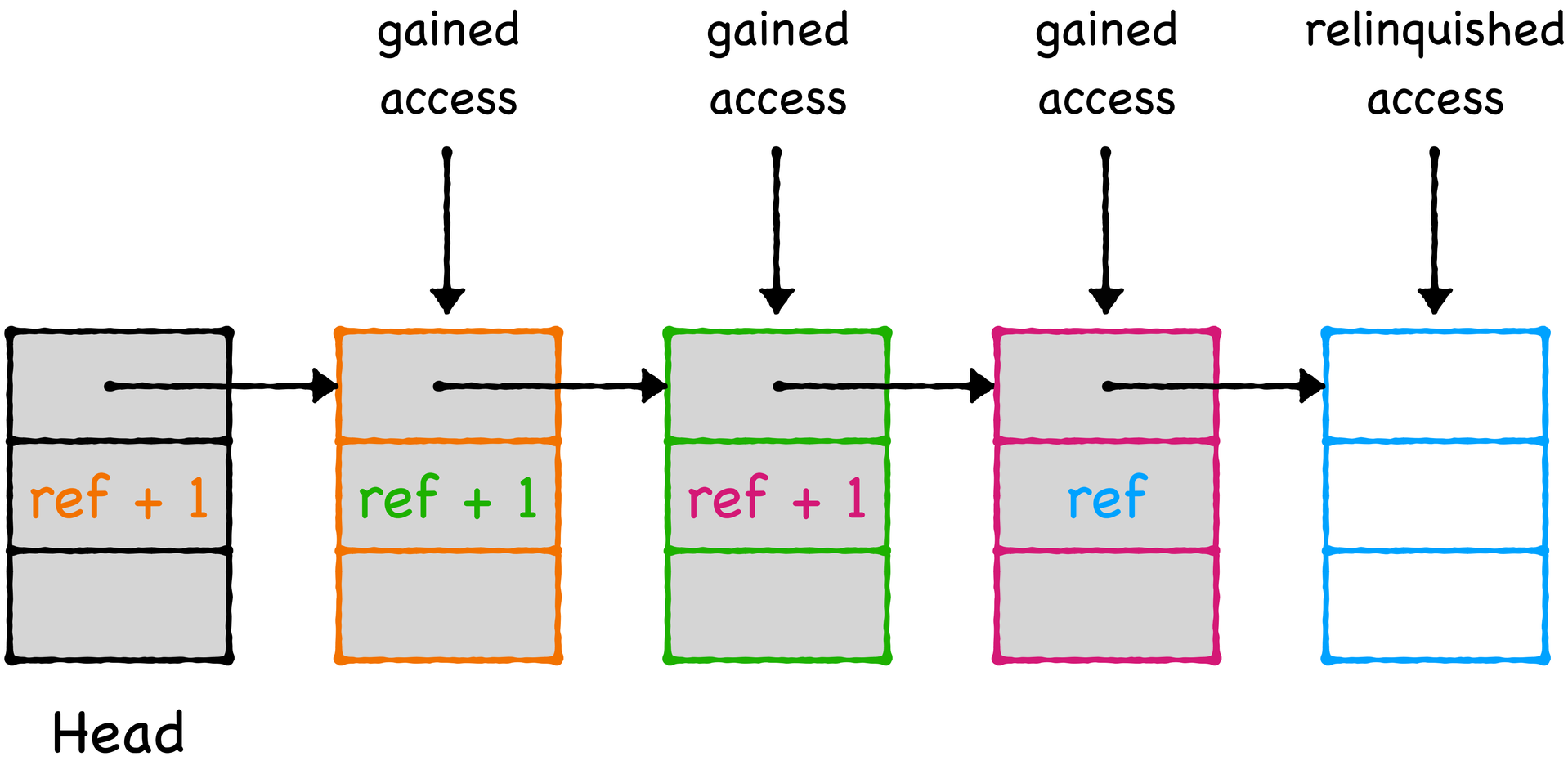
Observe: at this point p has the "right" to access every cell



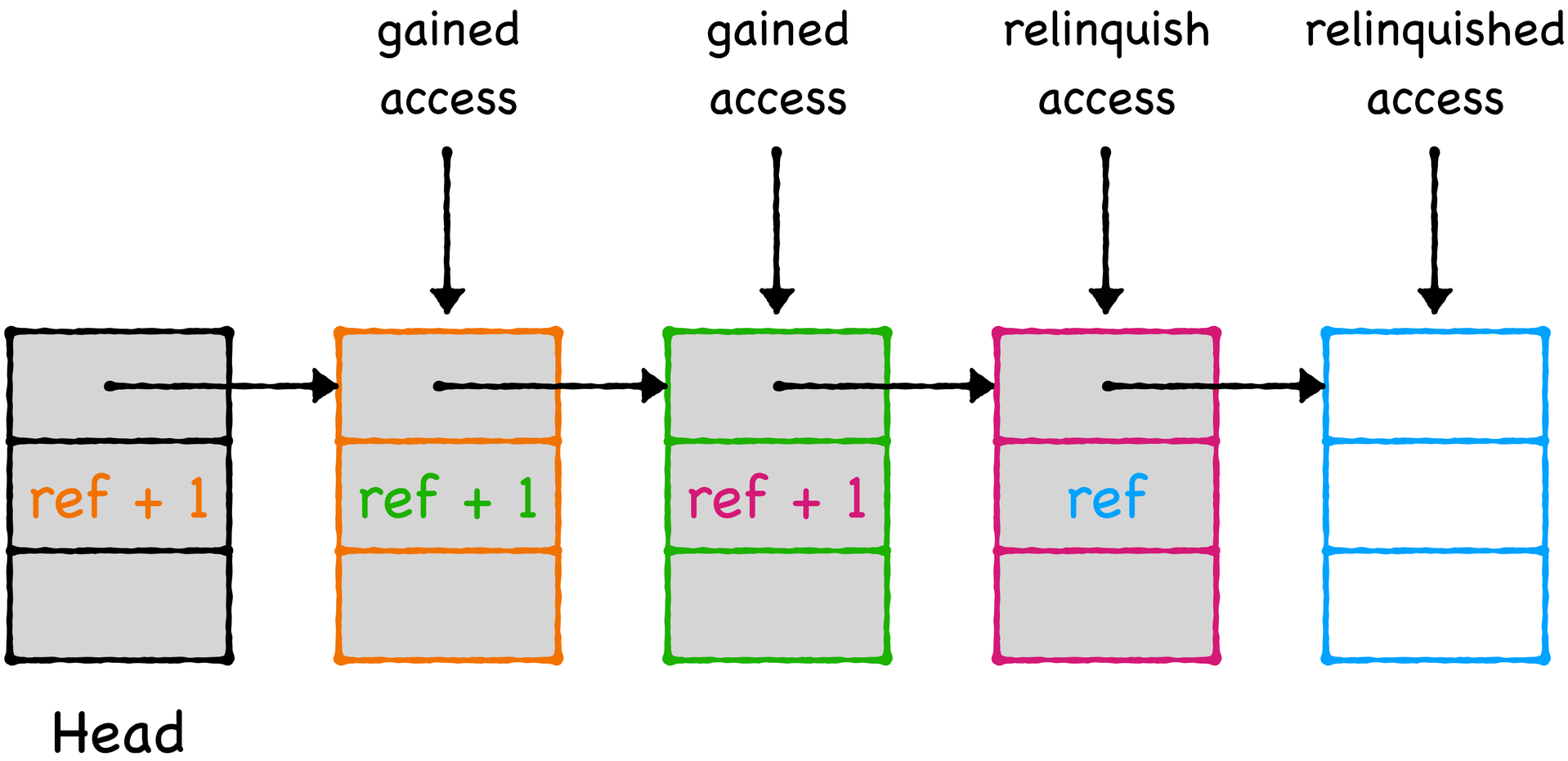
# of acquired cells: 5



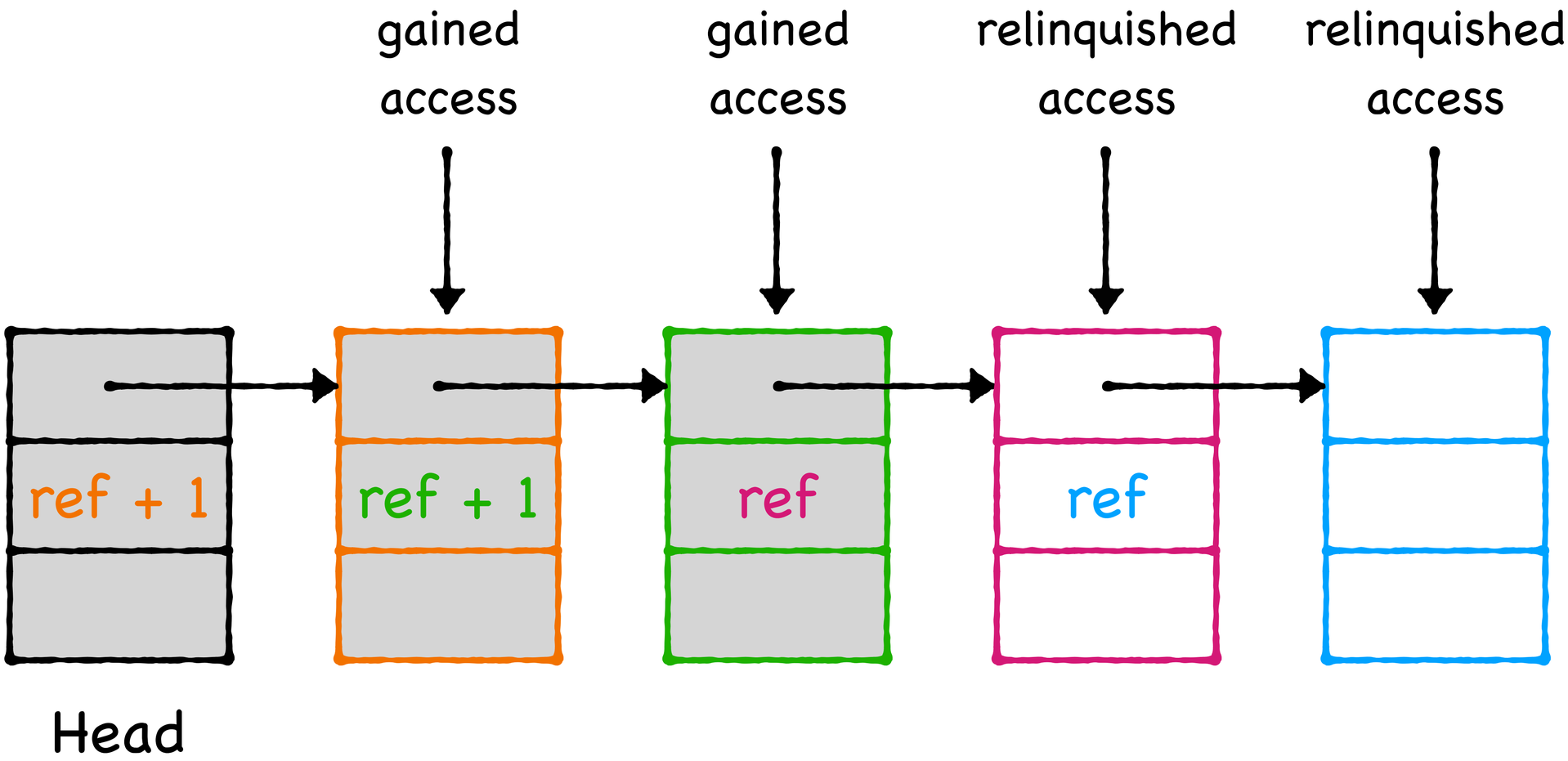
# of acquired cells: 4



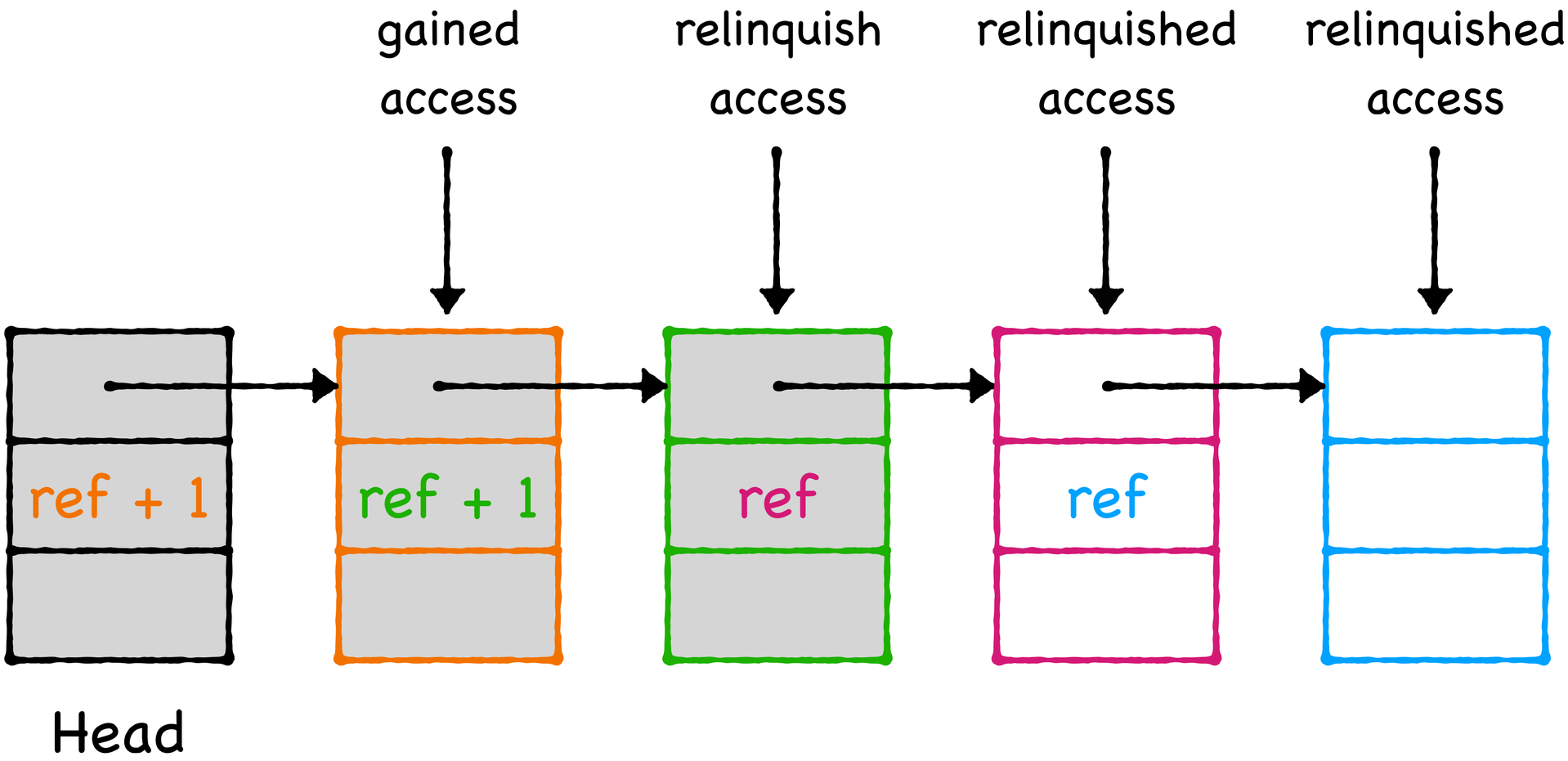
# of acquired cells: 4



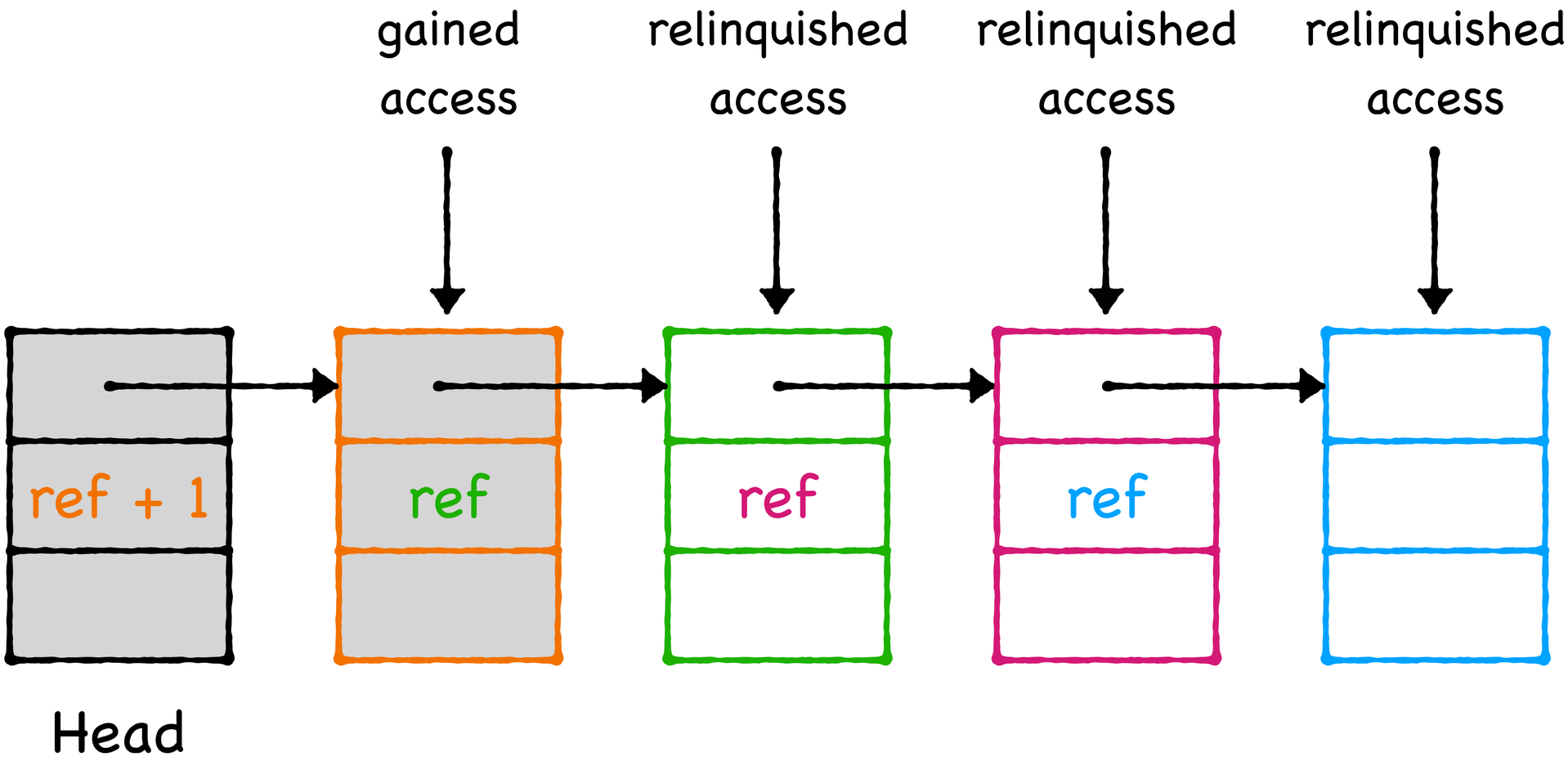
# of acquired cells: 4



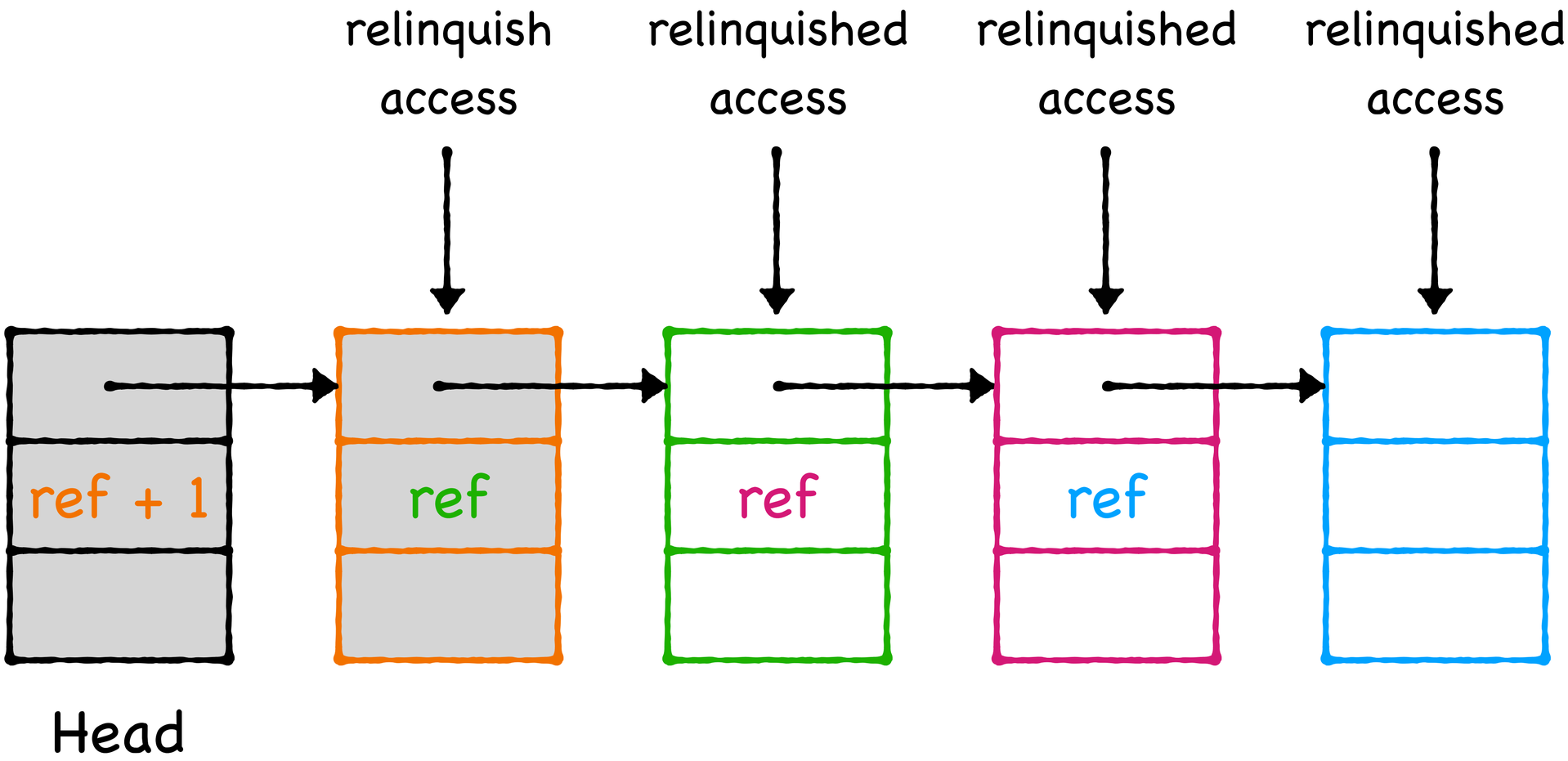
# of acquired cells: 3



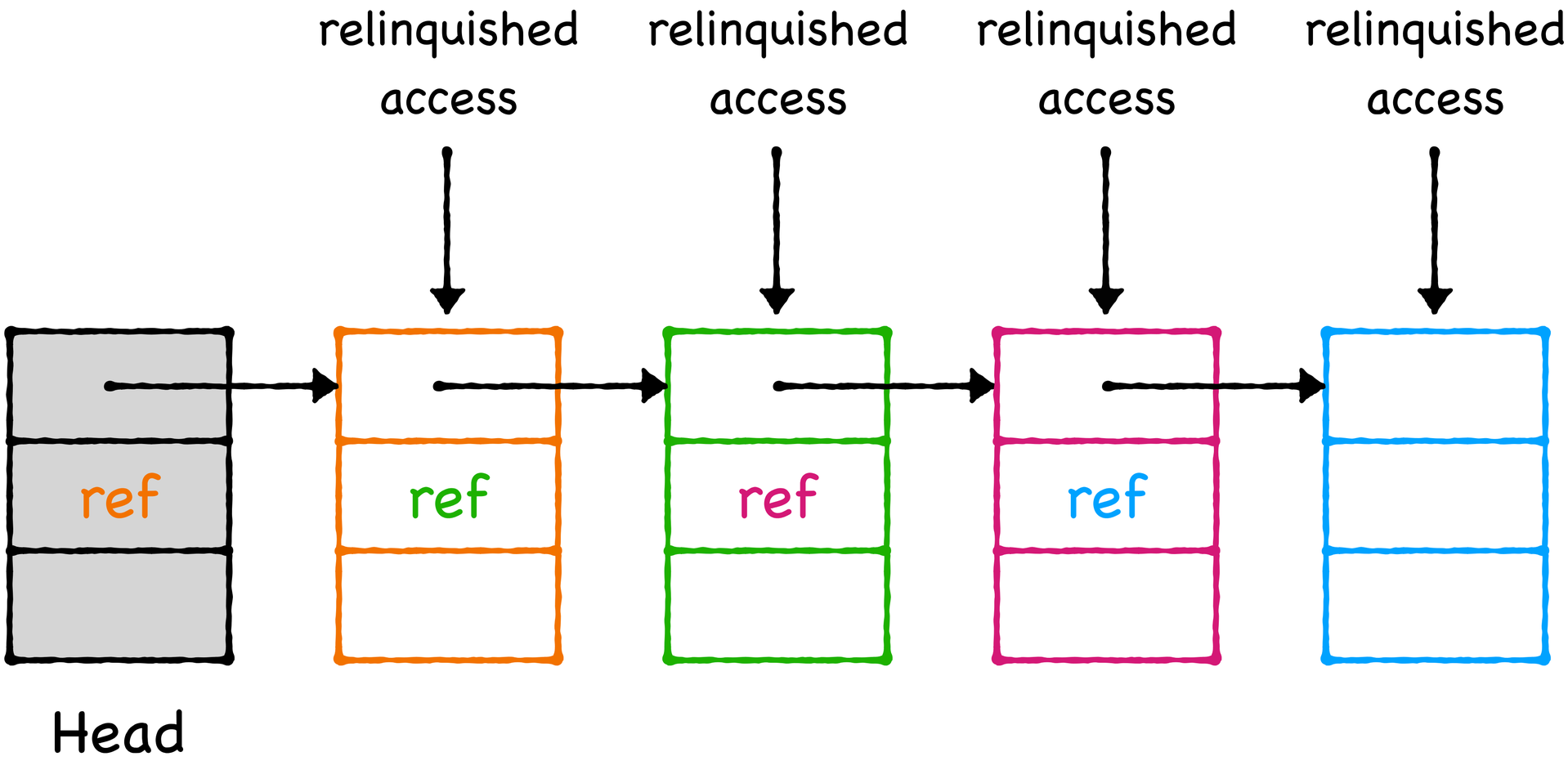
# of acquired cells: 3



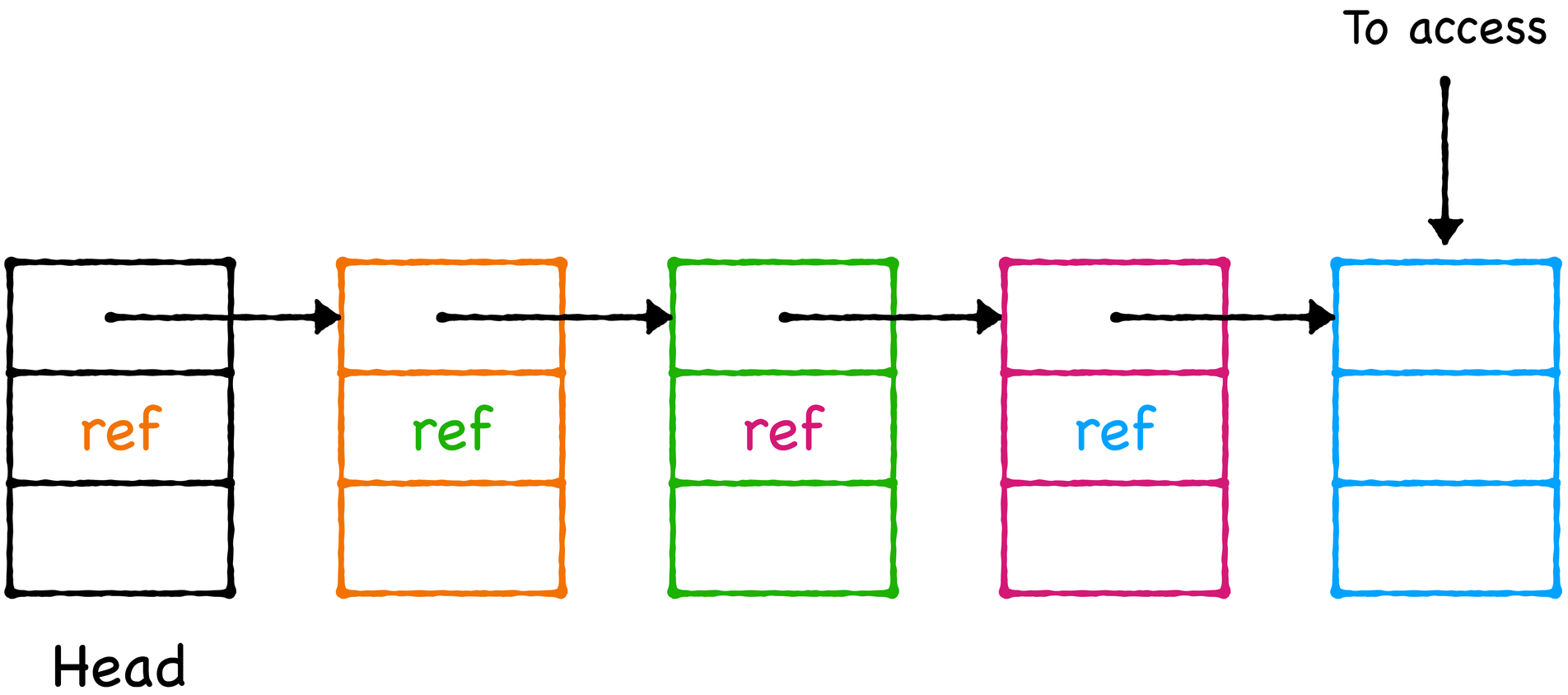
# of acquired cells: 2



# of acquired cells: 2



# of acquired cells: 1


To access
ref
ref
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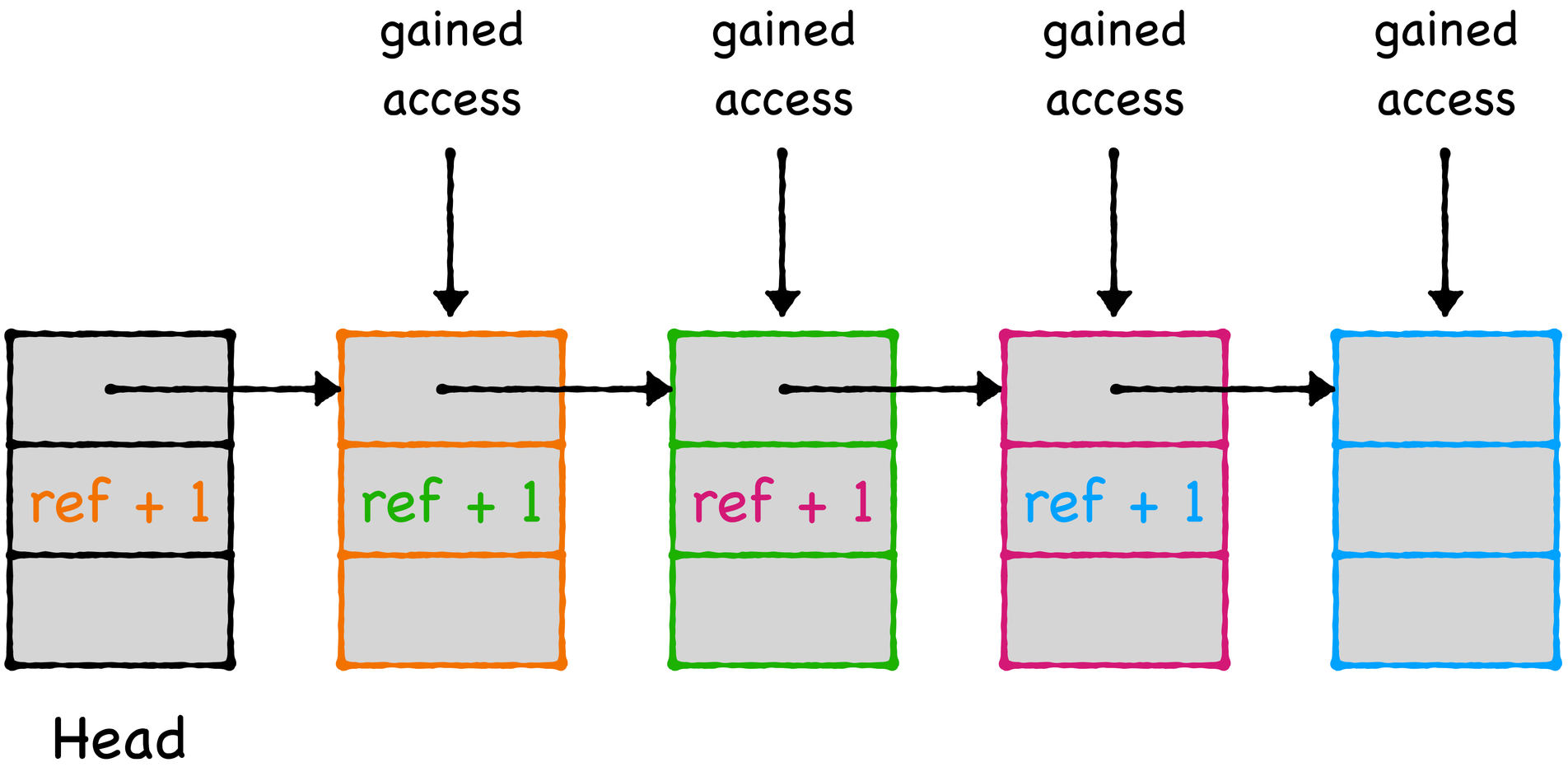
Head


gained access
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ref + 1
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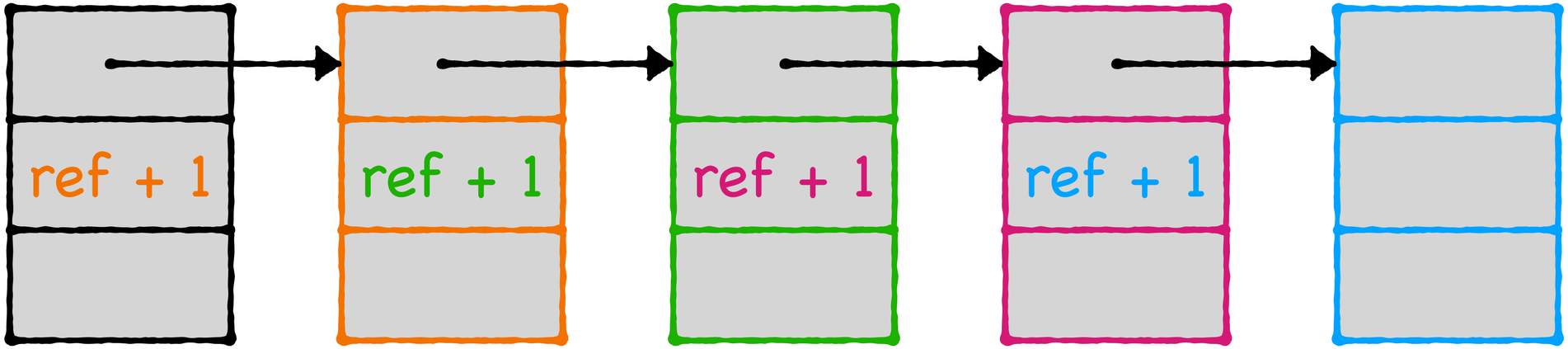
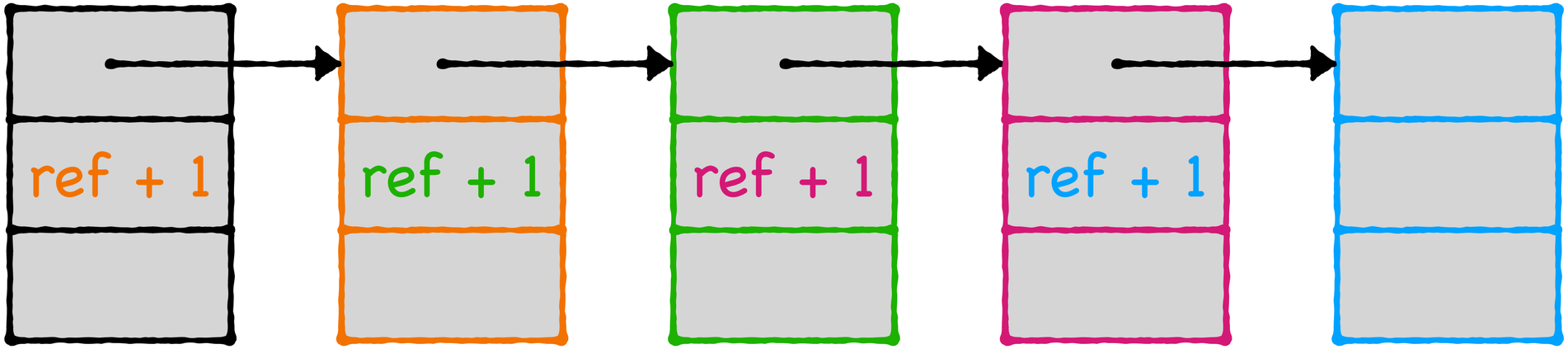


But what if p crashes when it has the **"right" to access every cell?**



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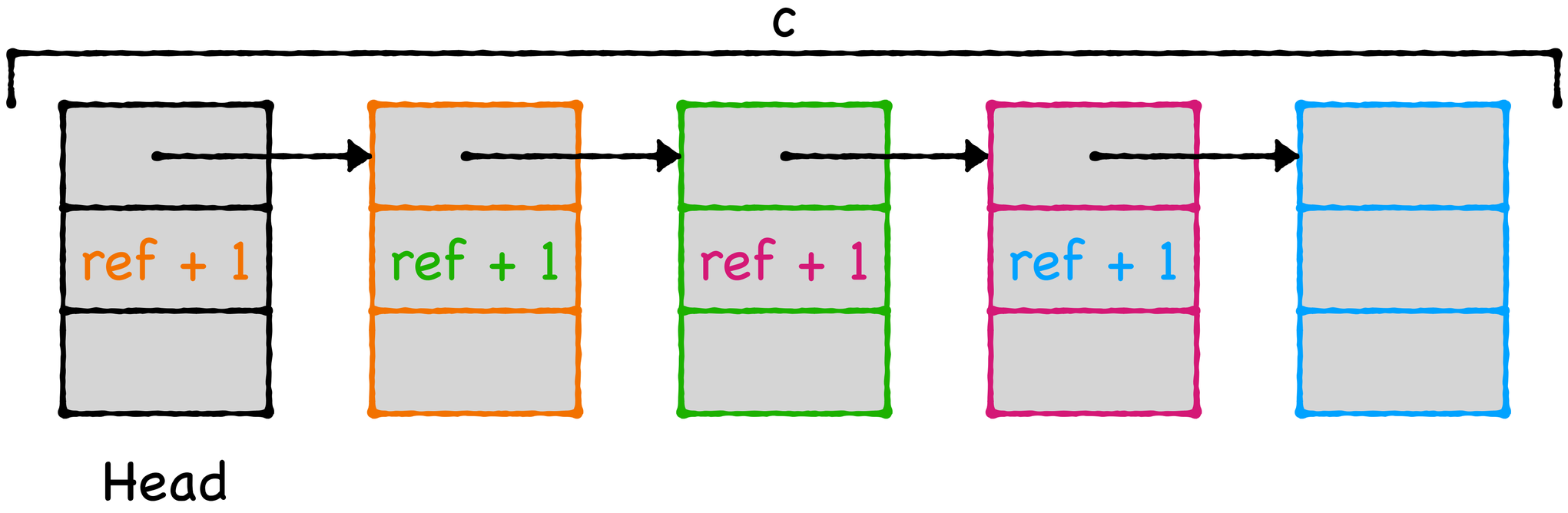

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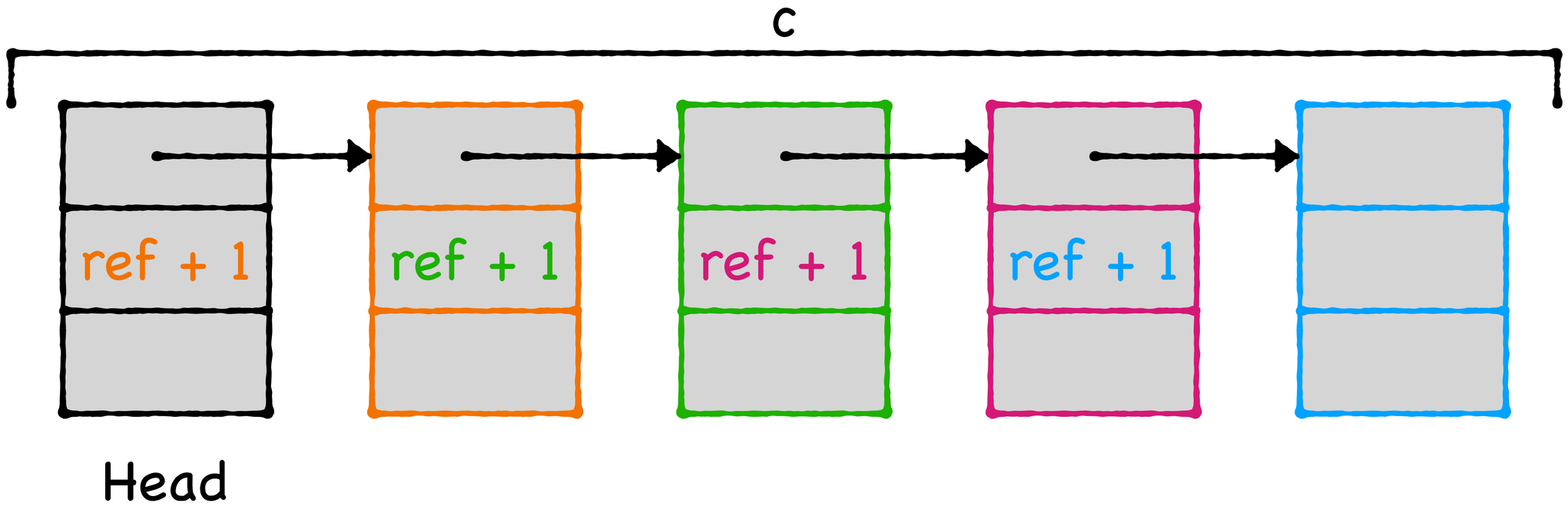


But what if p crashes when it has the **"right" to access every cell?**

None of these cells **can ever be recycled!**

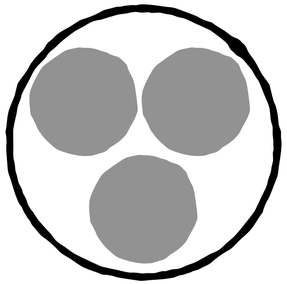
The space complexity is $\Omega(c^2)$ where c is the point contention

Improving the Space Complexity

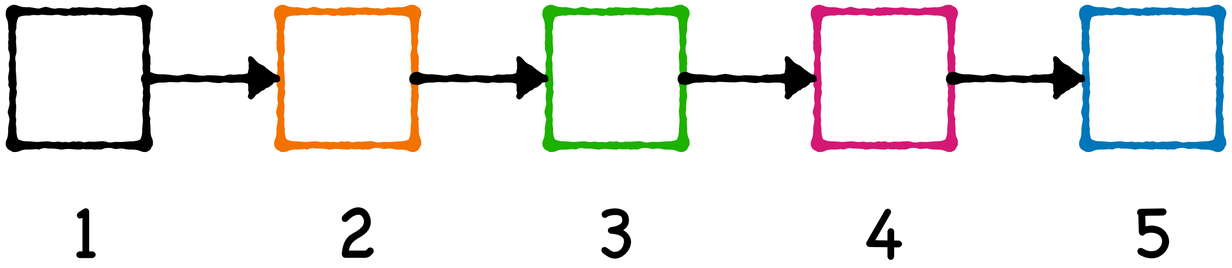
Goal: reduce the number of cells any process has the right to access at any point in time

Yes!

The Reversible Pebble Game [Bennett'89]



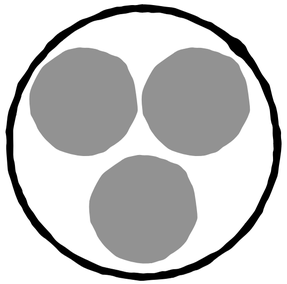
Pebbles



Given:

(1) A list of n nodes (here n = 5)

(2) A set of m pebbles (here m = 3)



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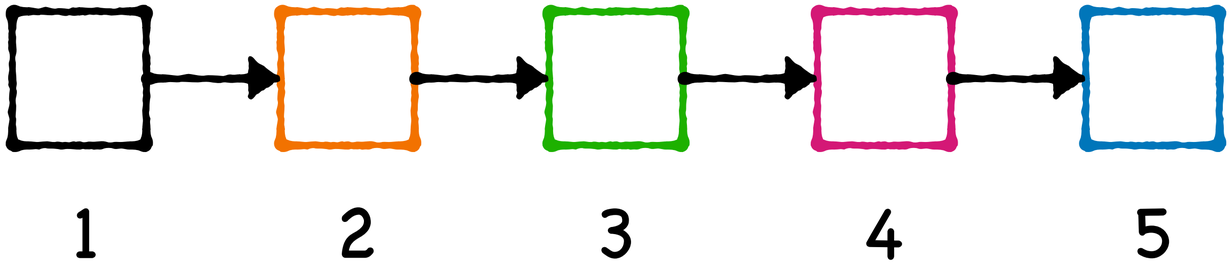

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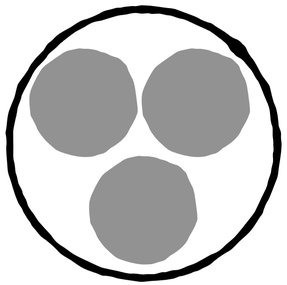

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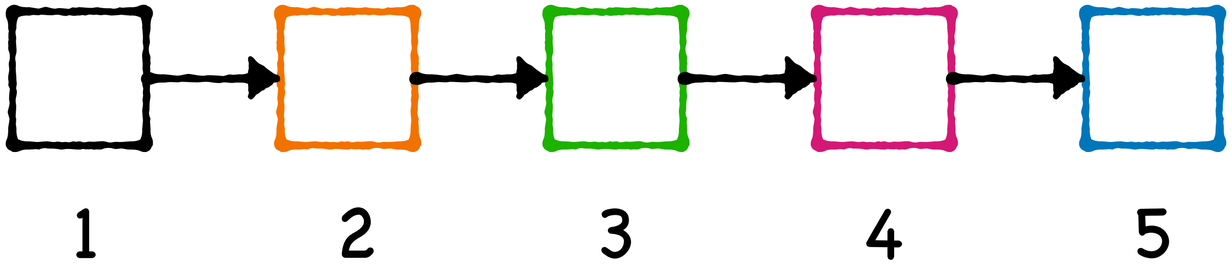

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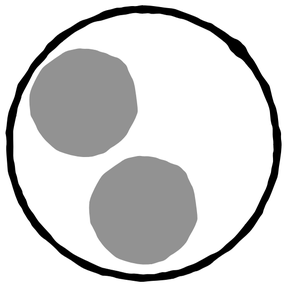

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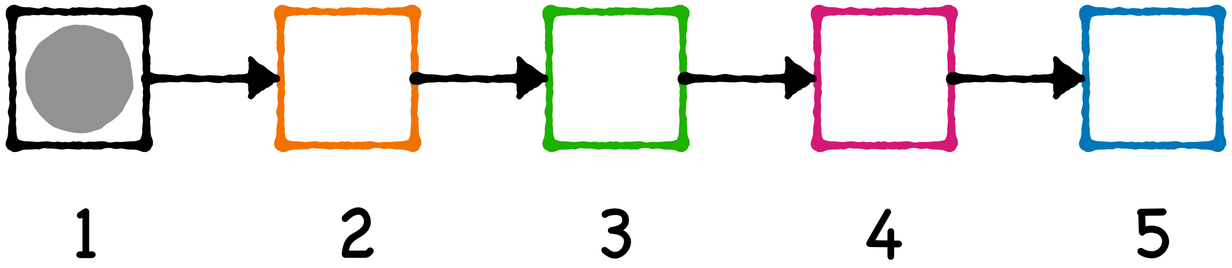

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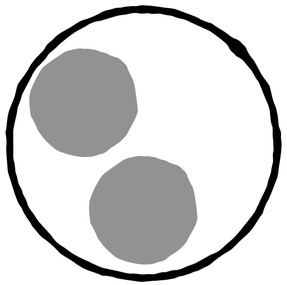

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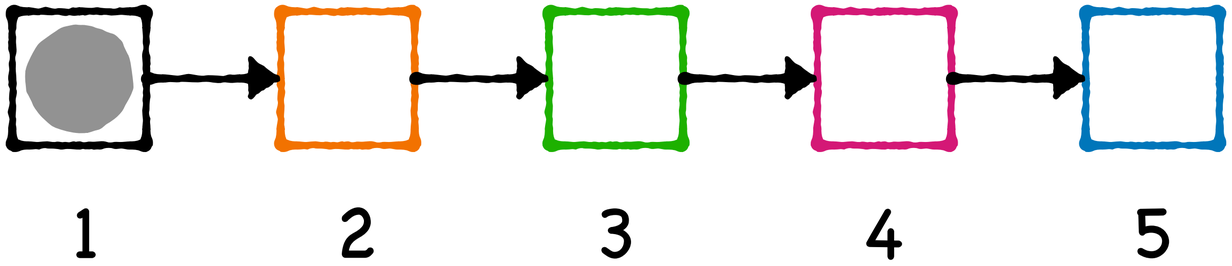

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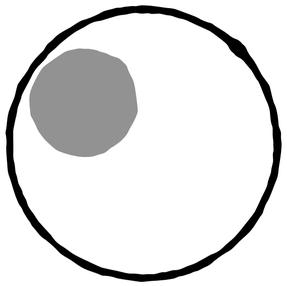

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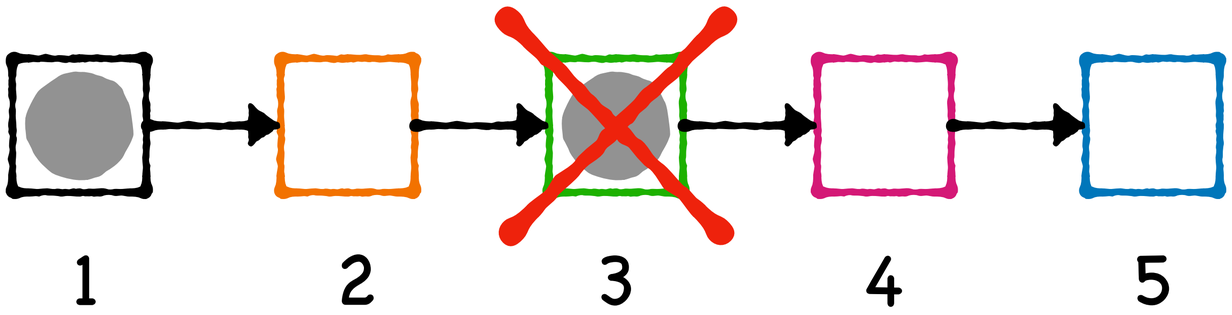

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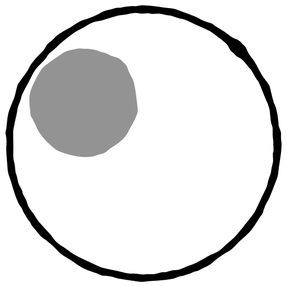

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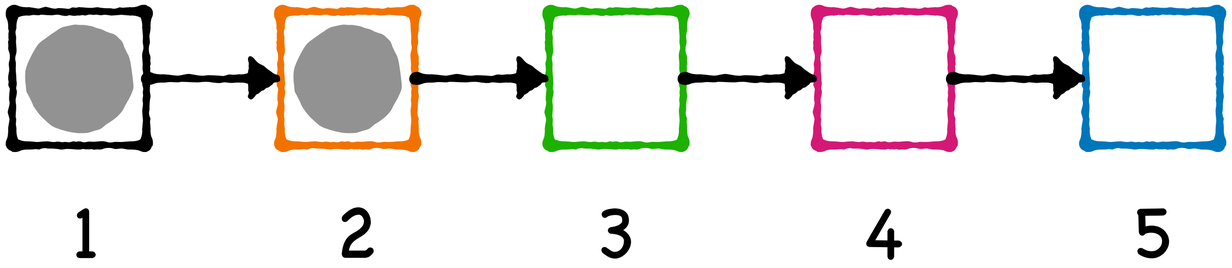

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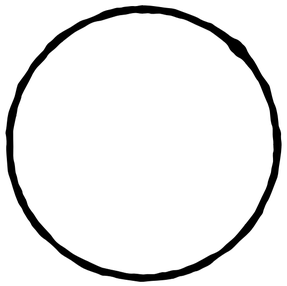

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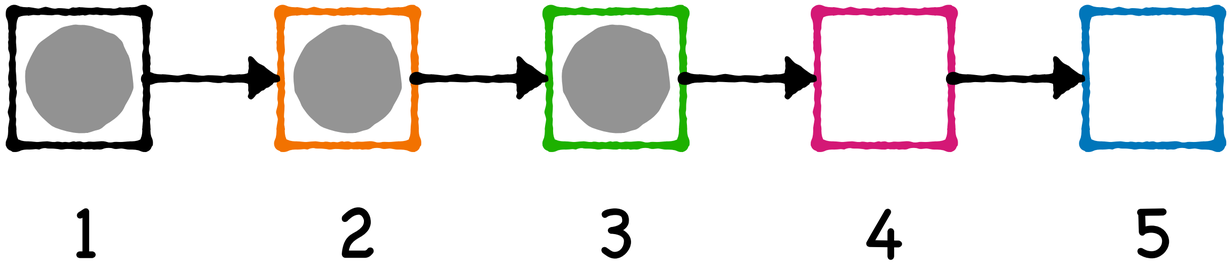

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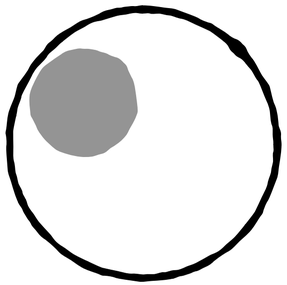

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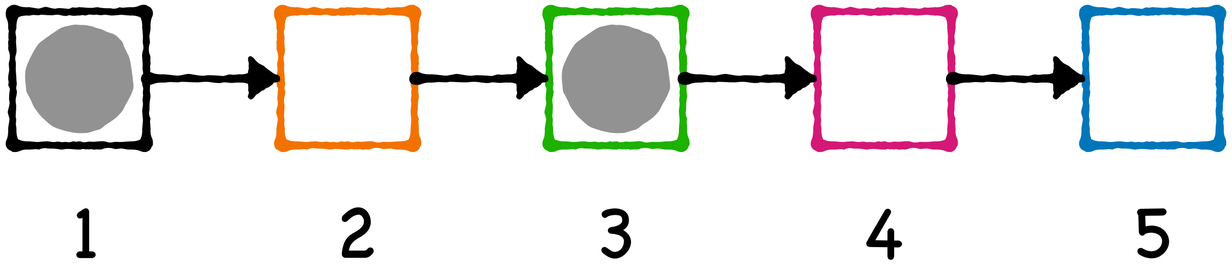

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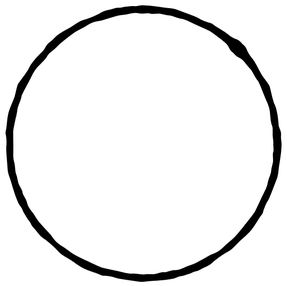

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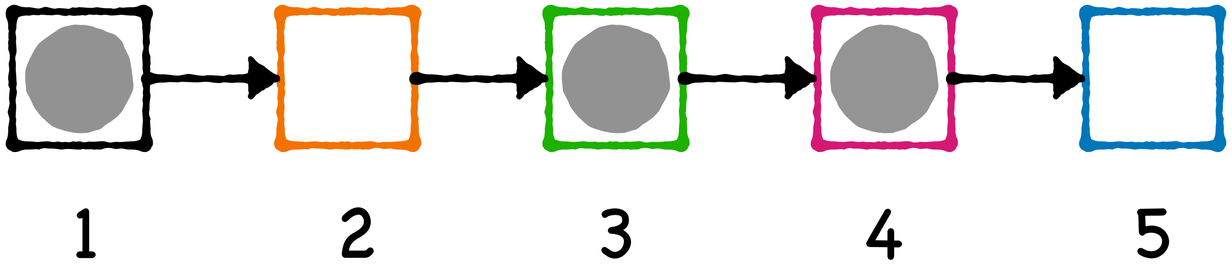

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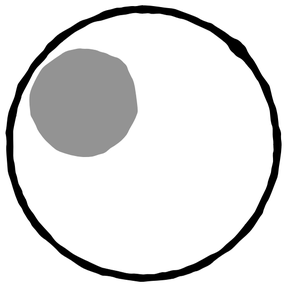

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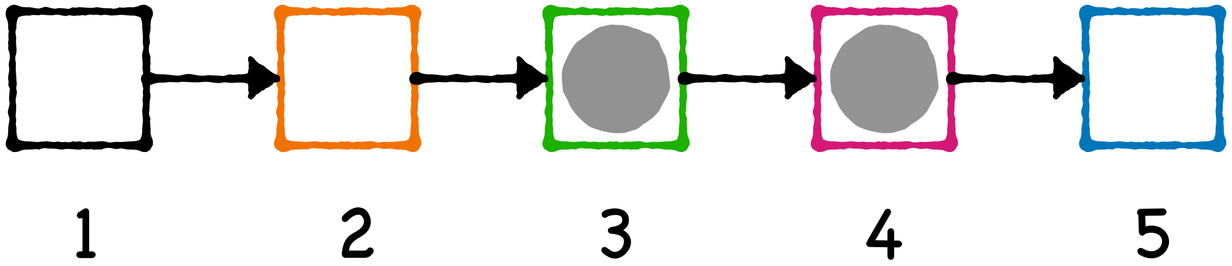

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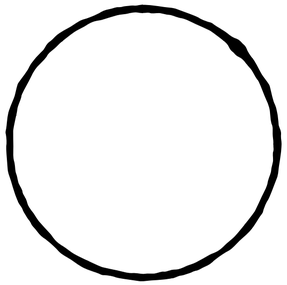

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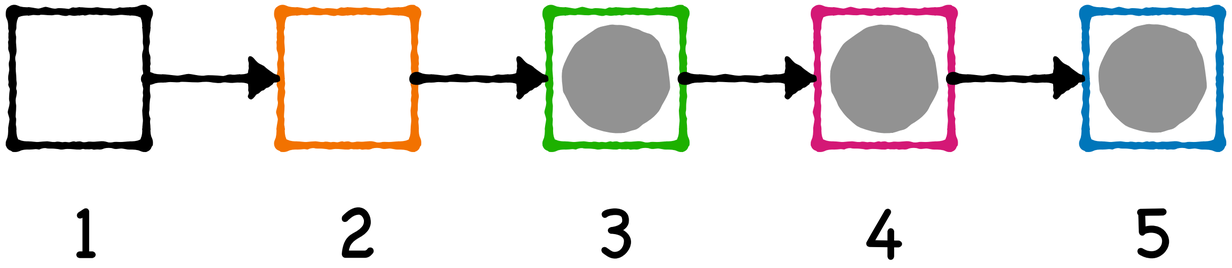

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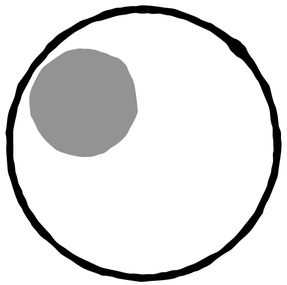

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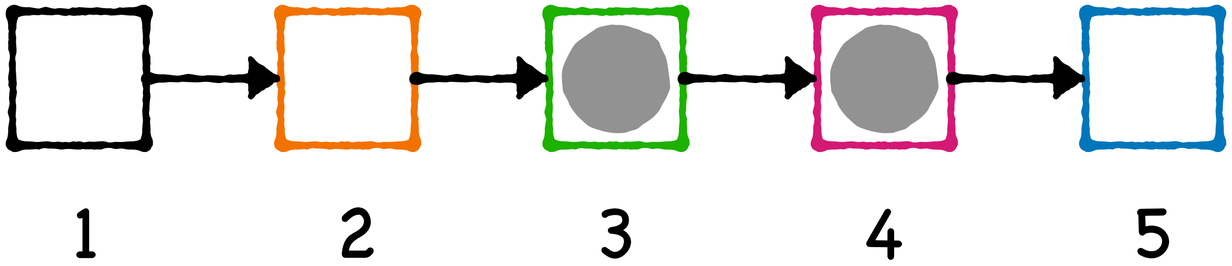

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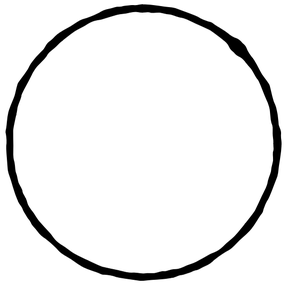

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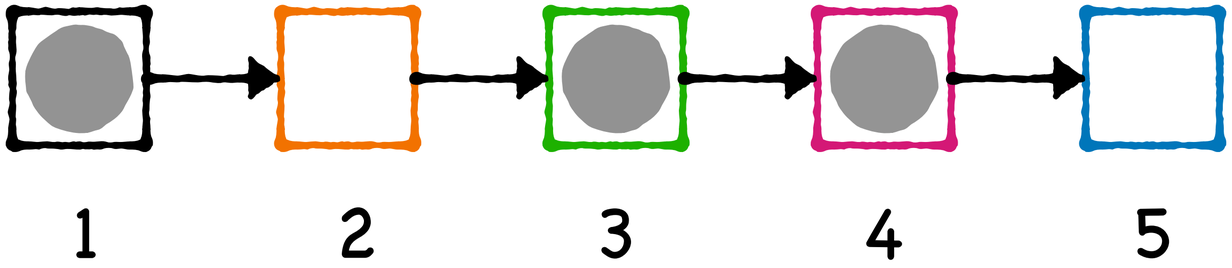

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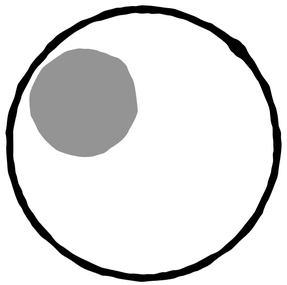

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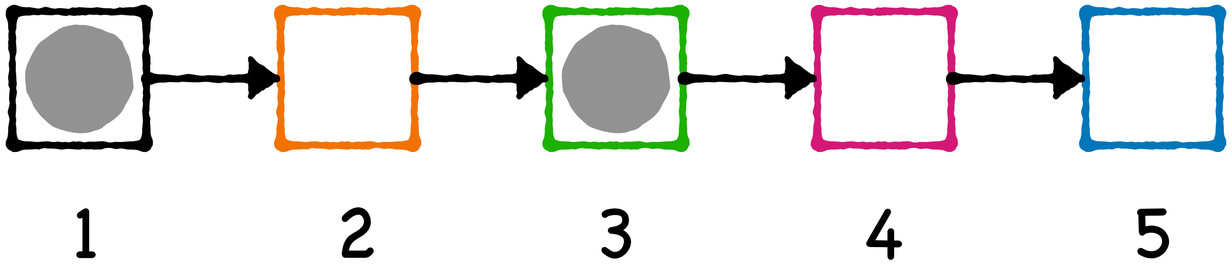

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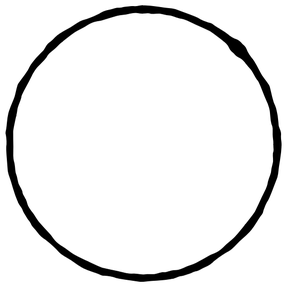

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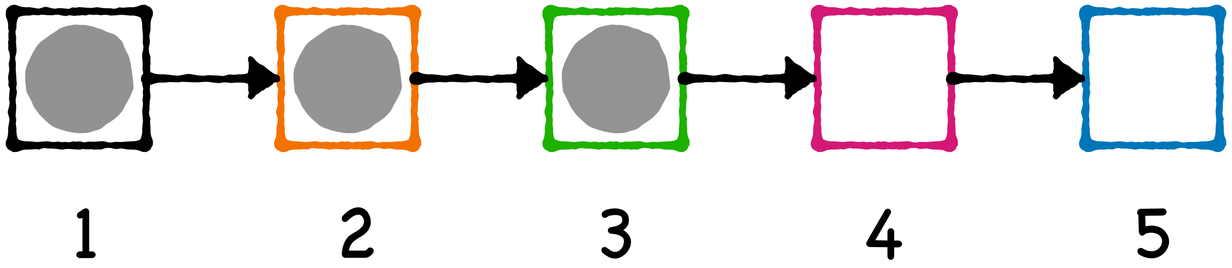

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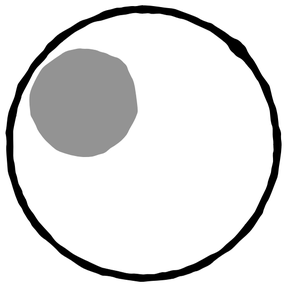

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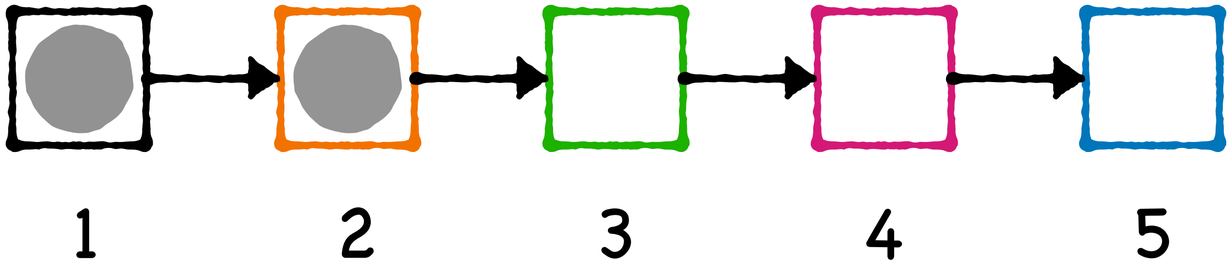

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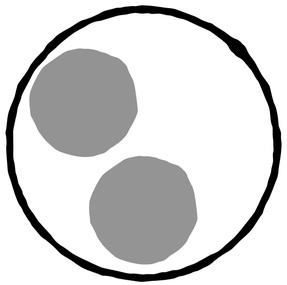

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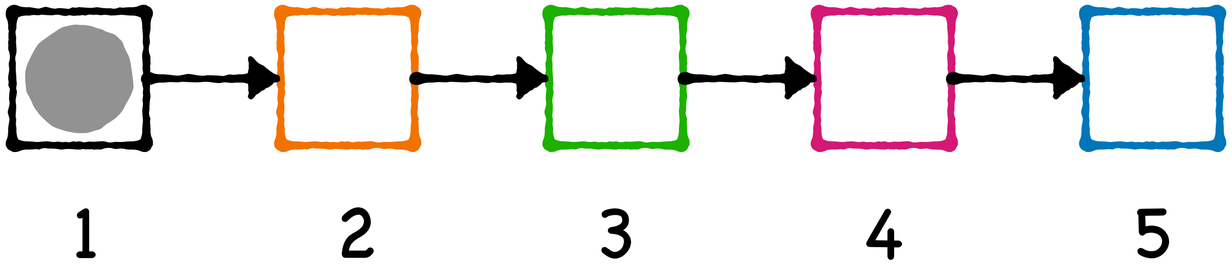

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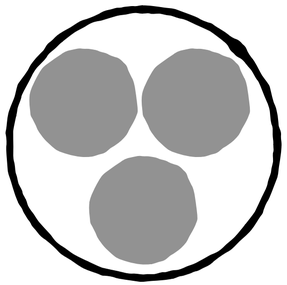

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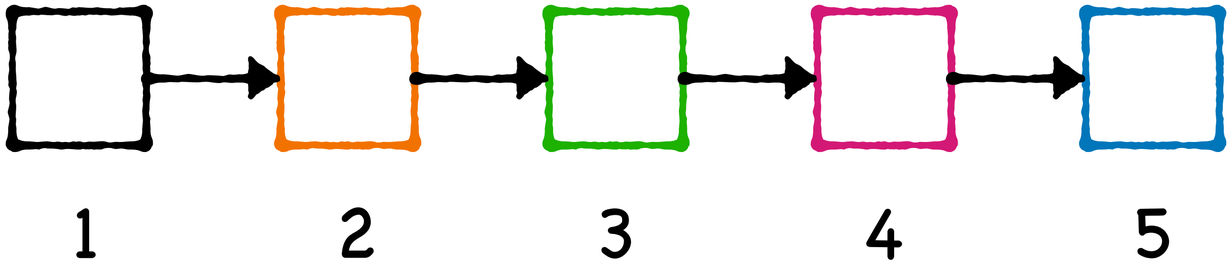

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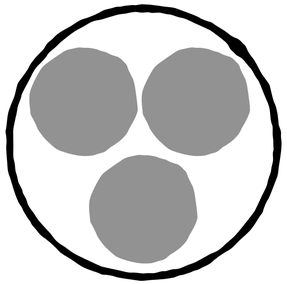

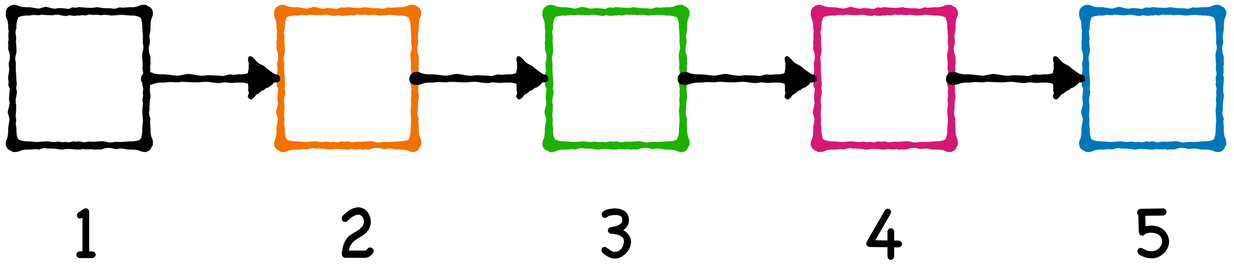

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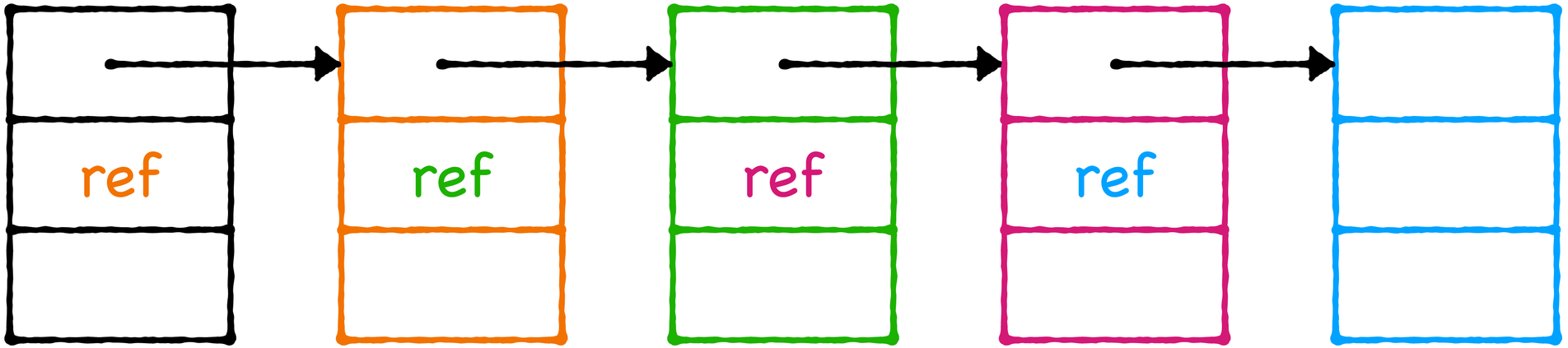

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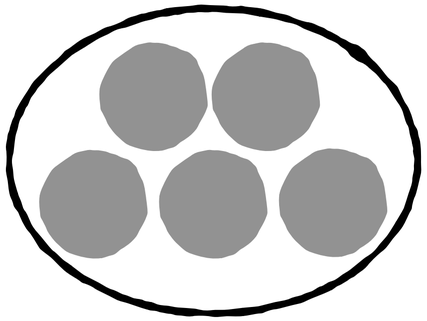
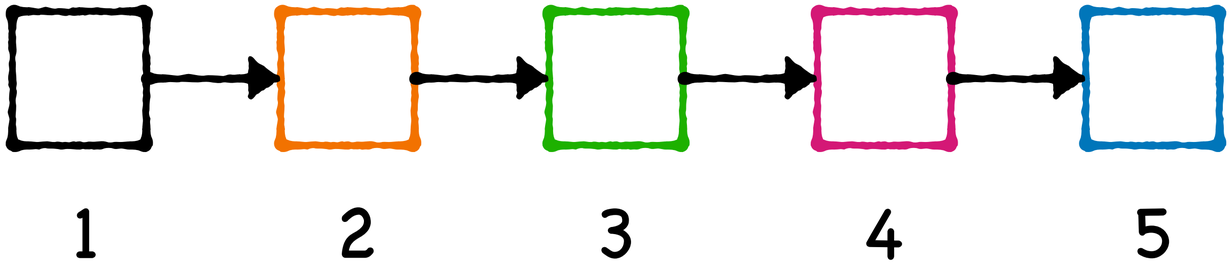
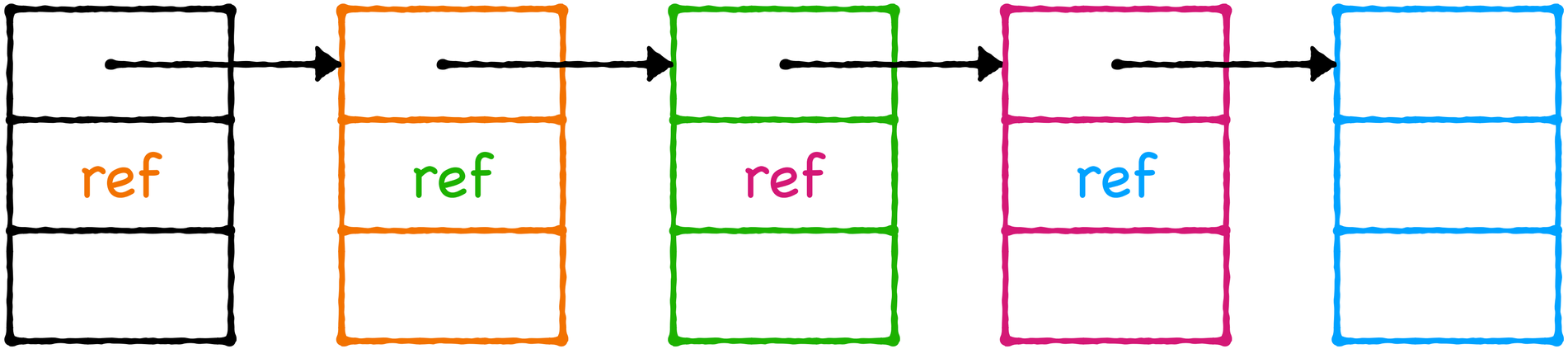


Pebbles





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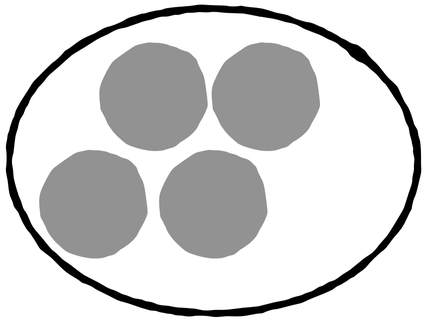

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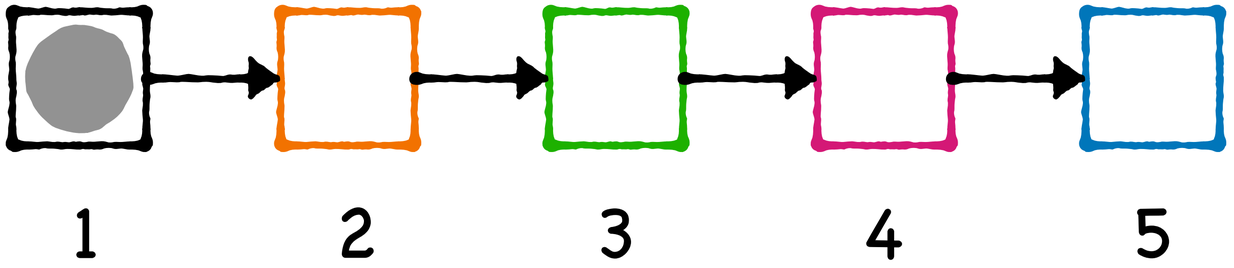

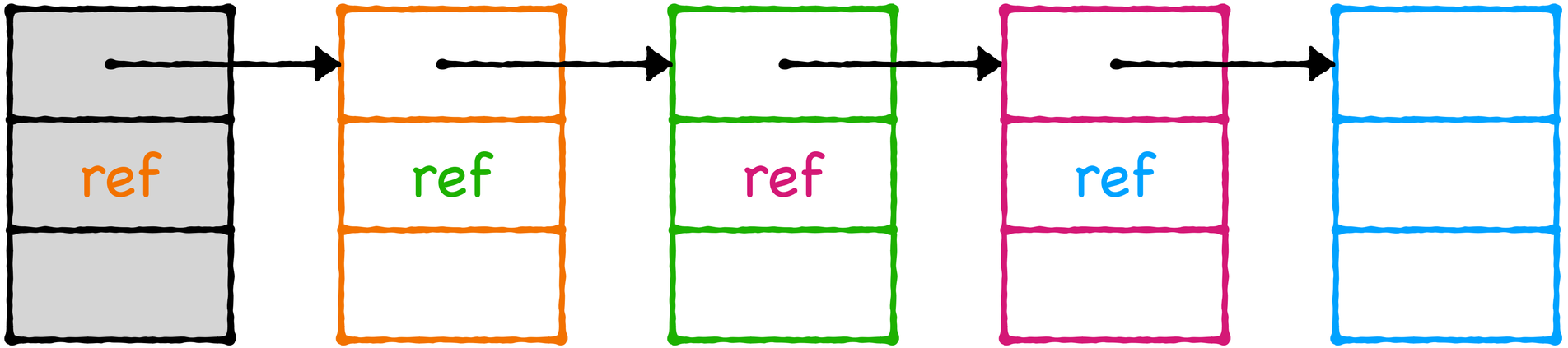

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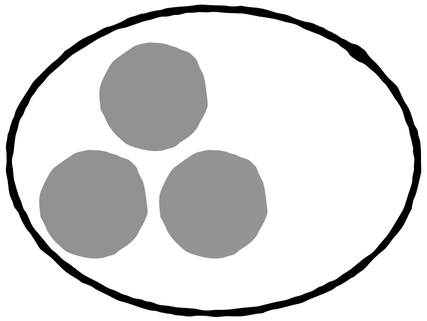

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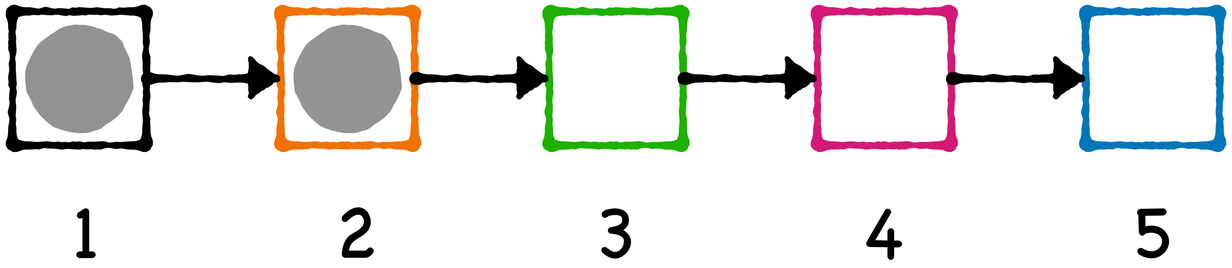

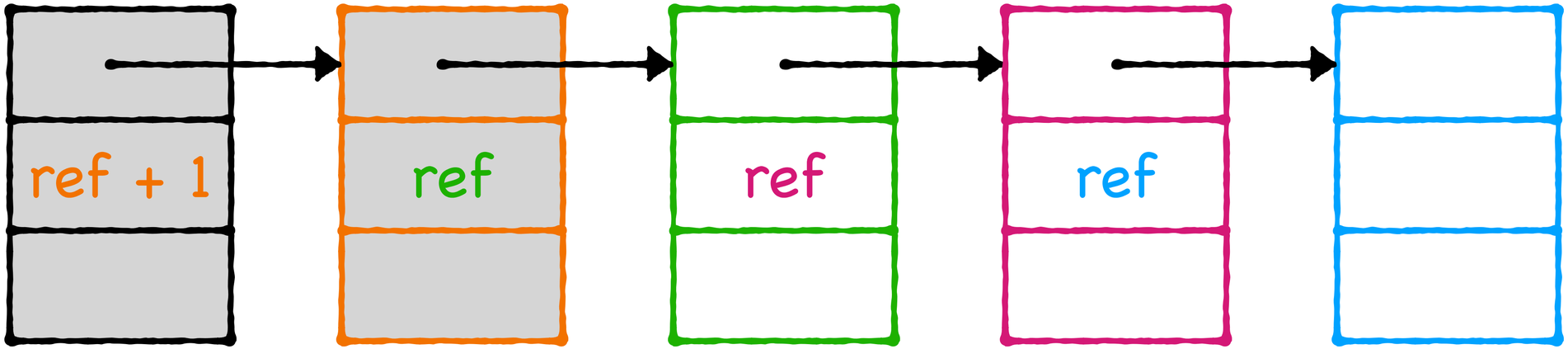

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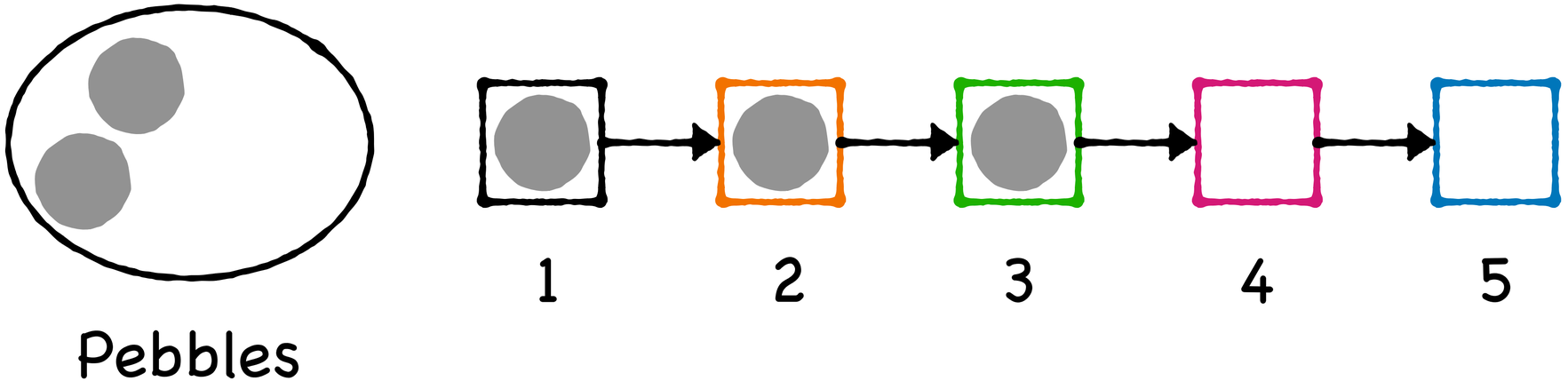

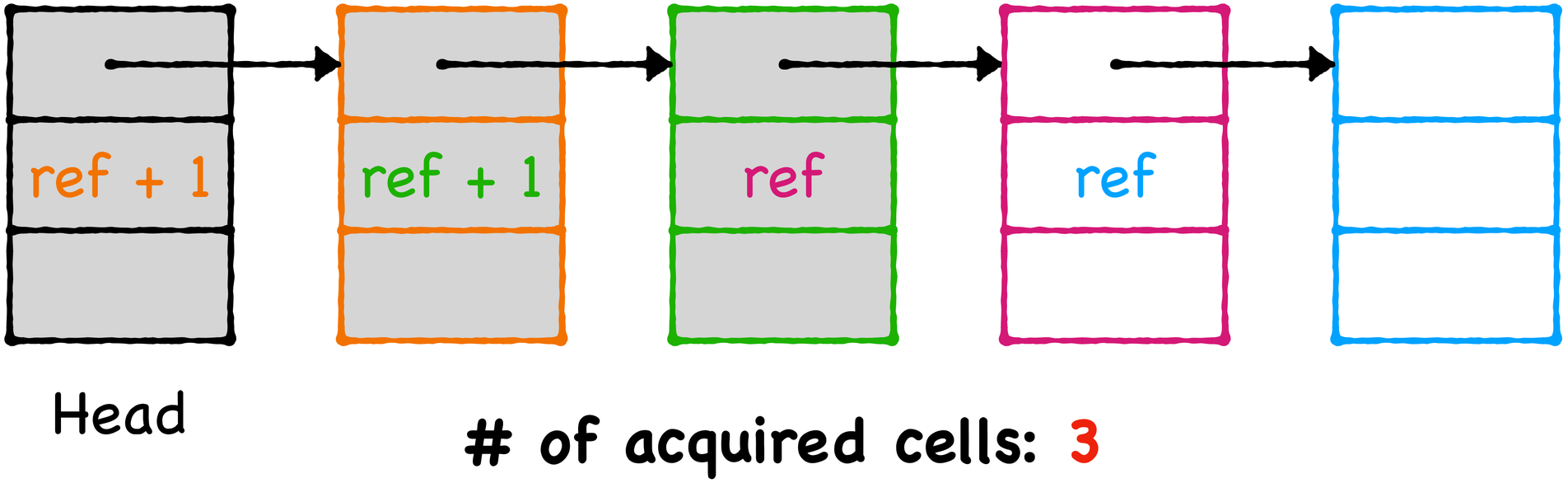

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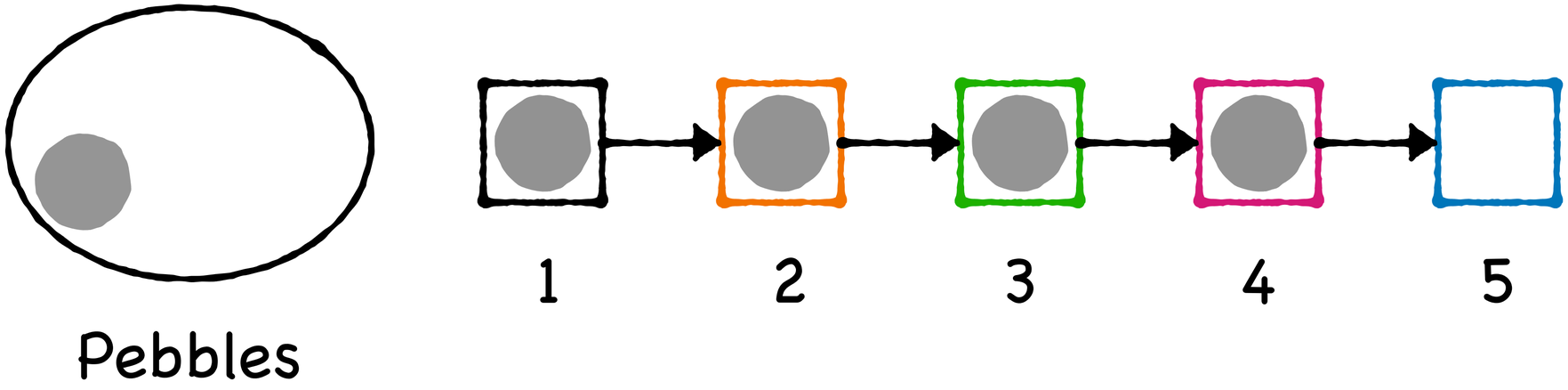

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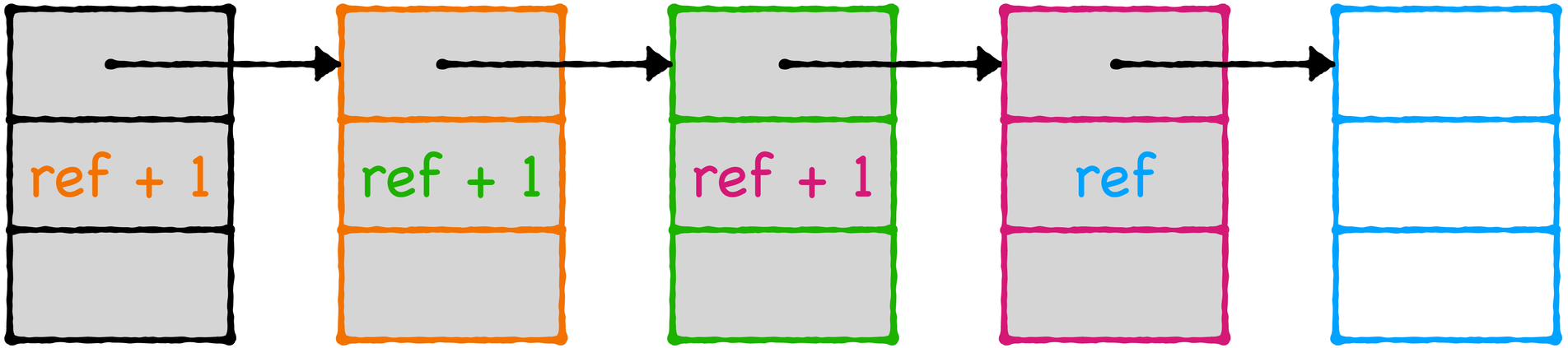

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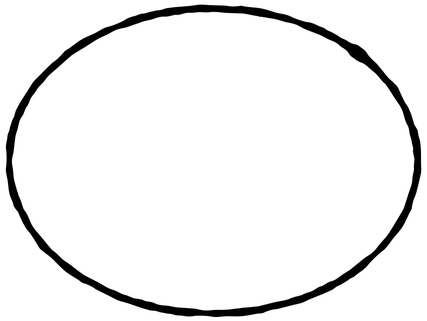

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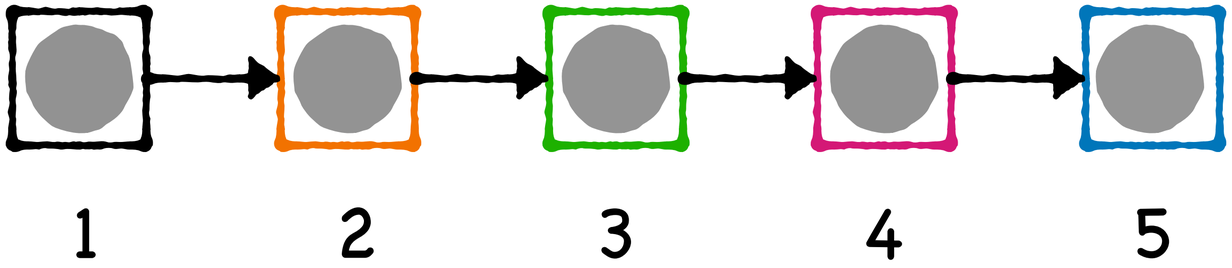

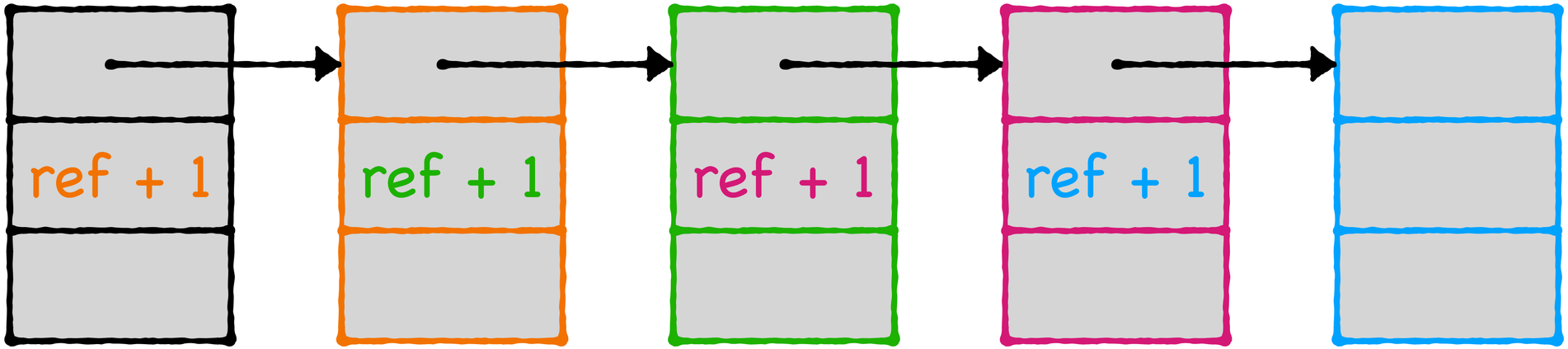

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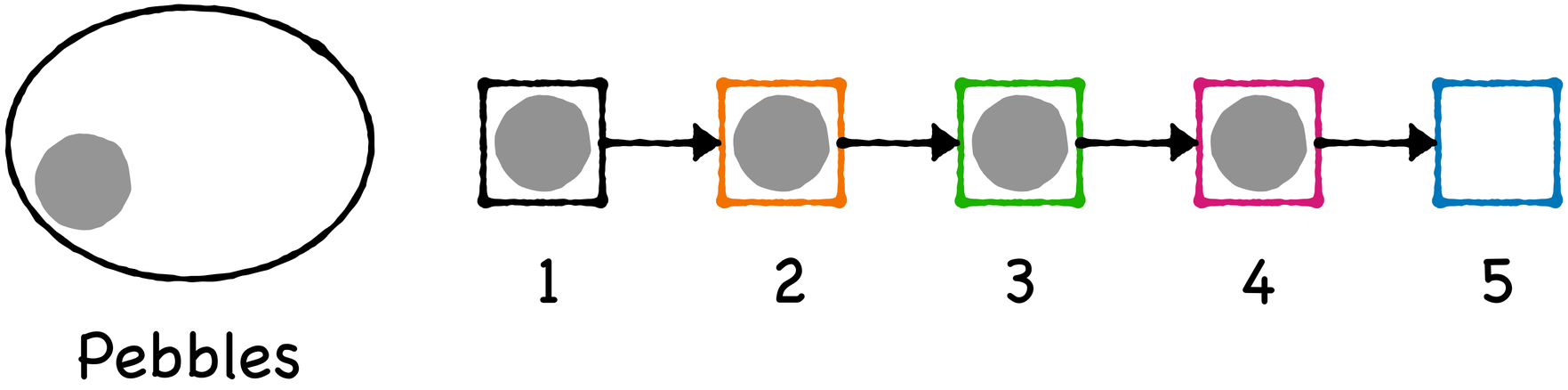

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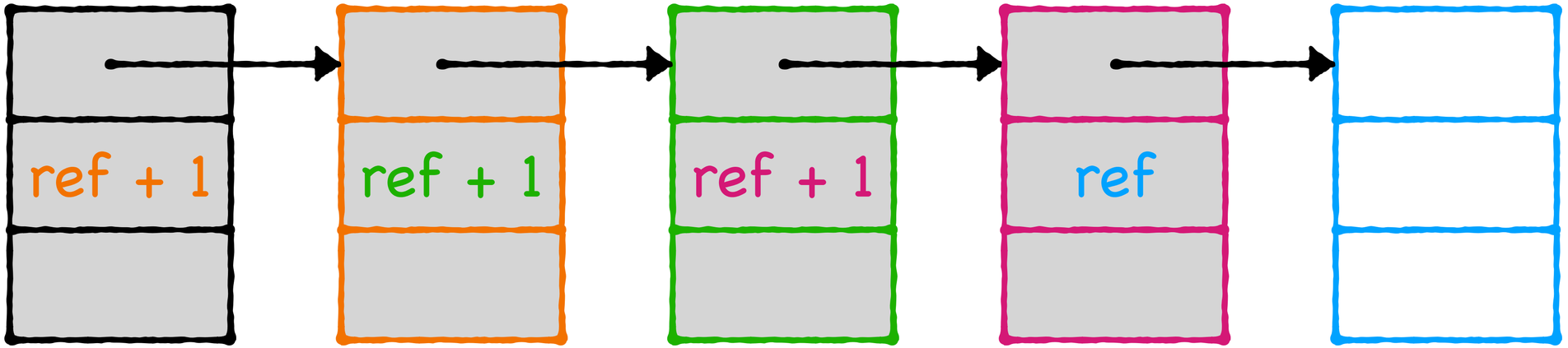

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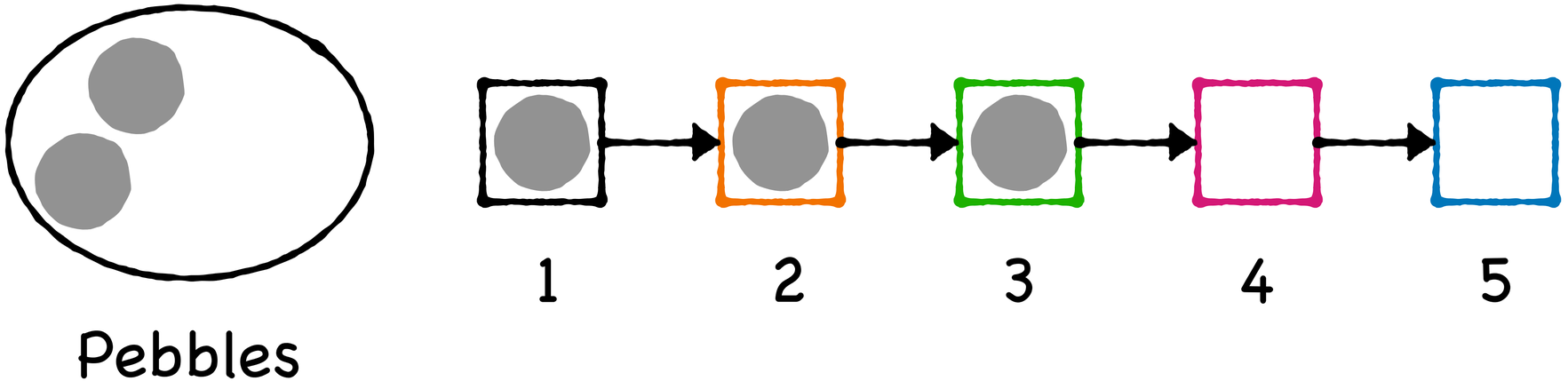

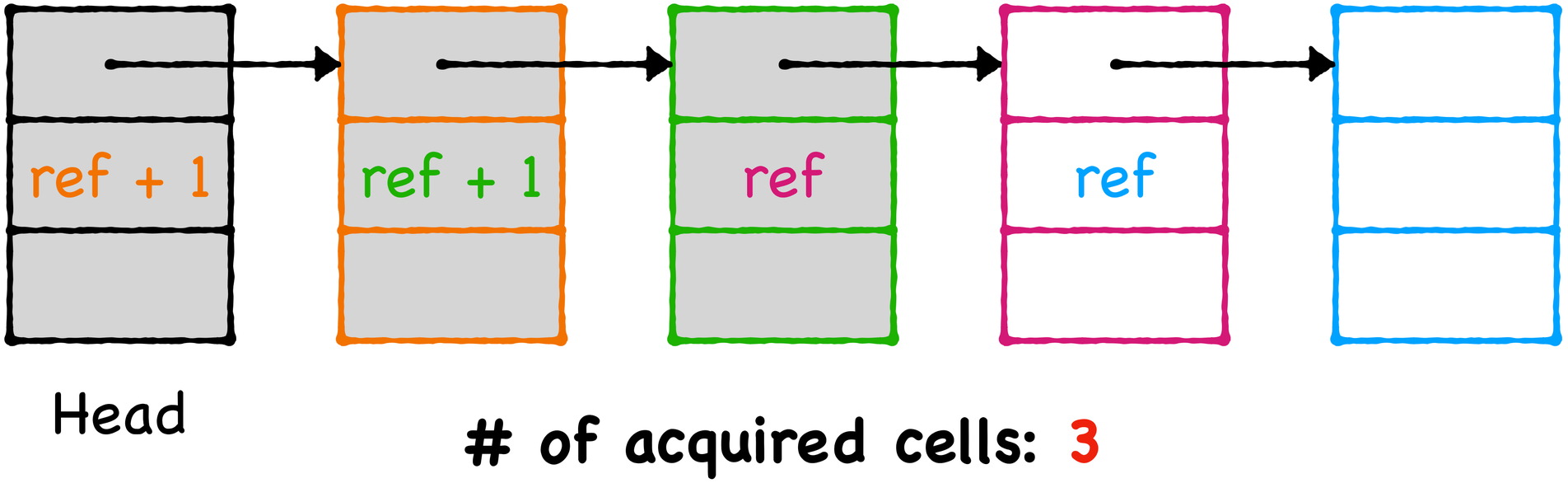

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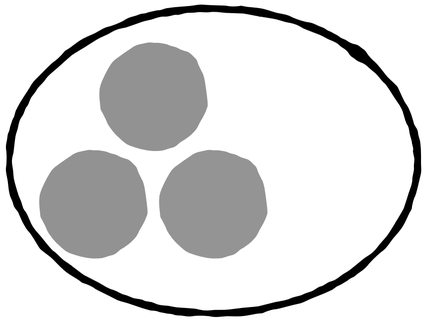

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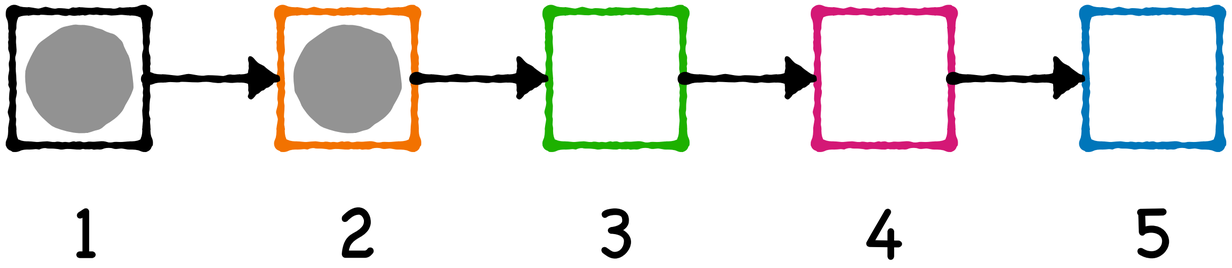

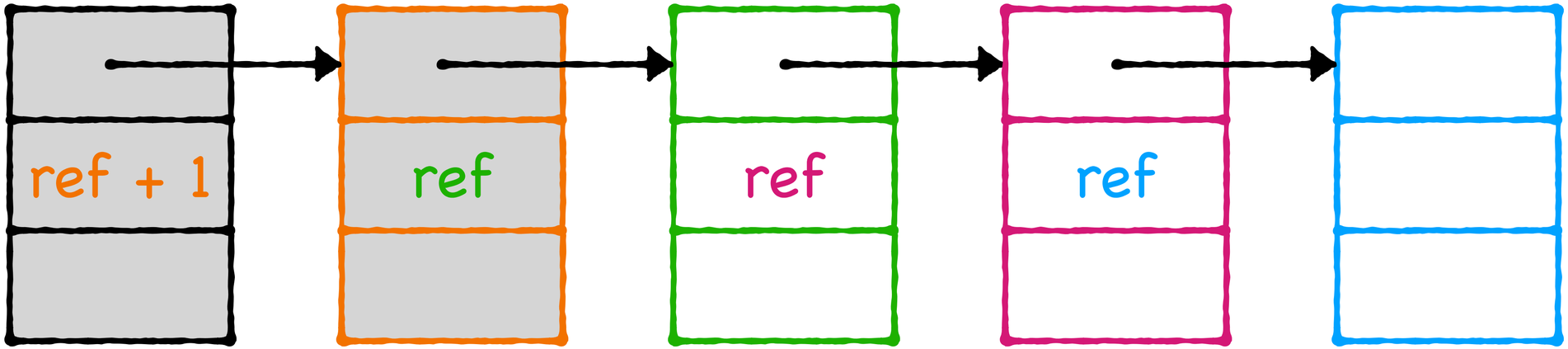

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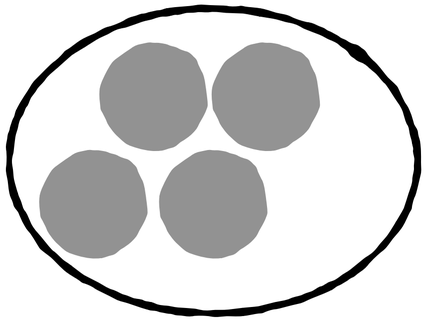

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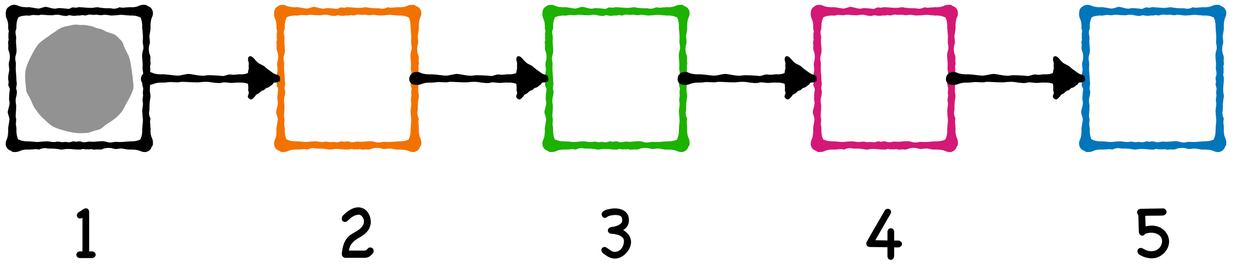

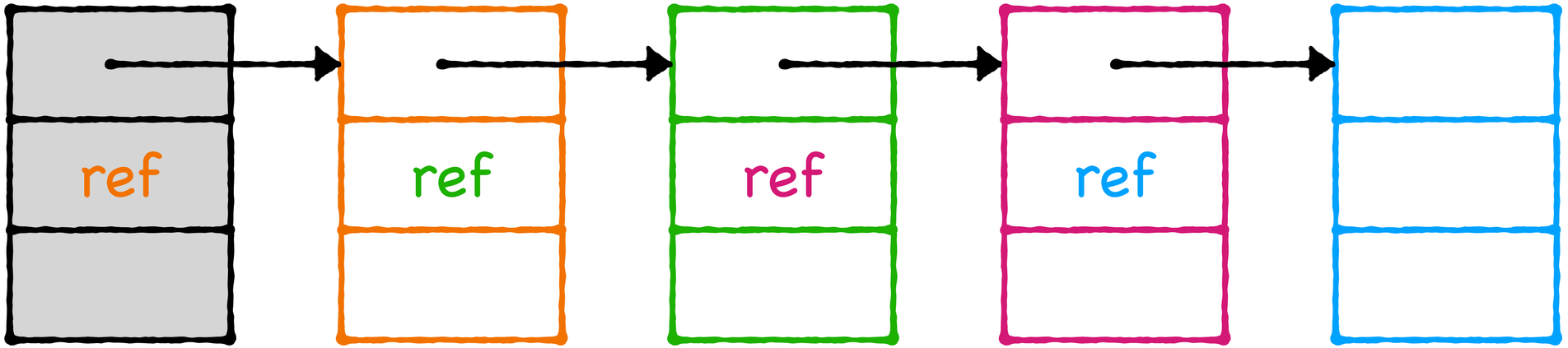

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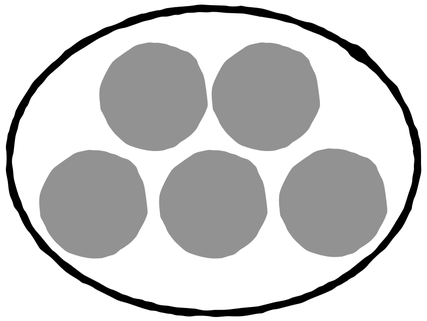

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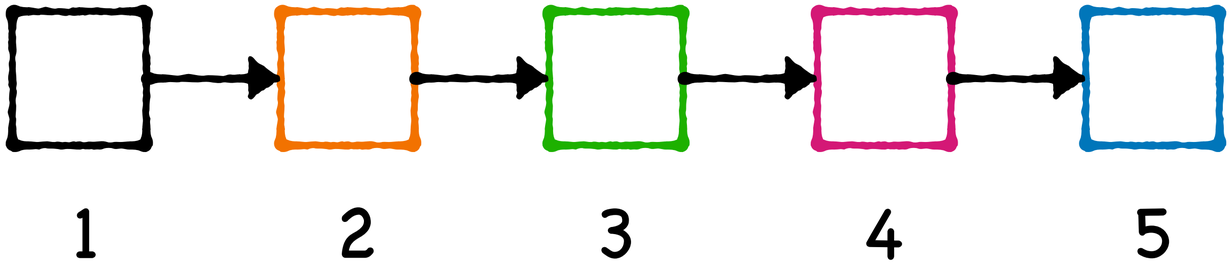

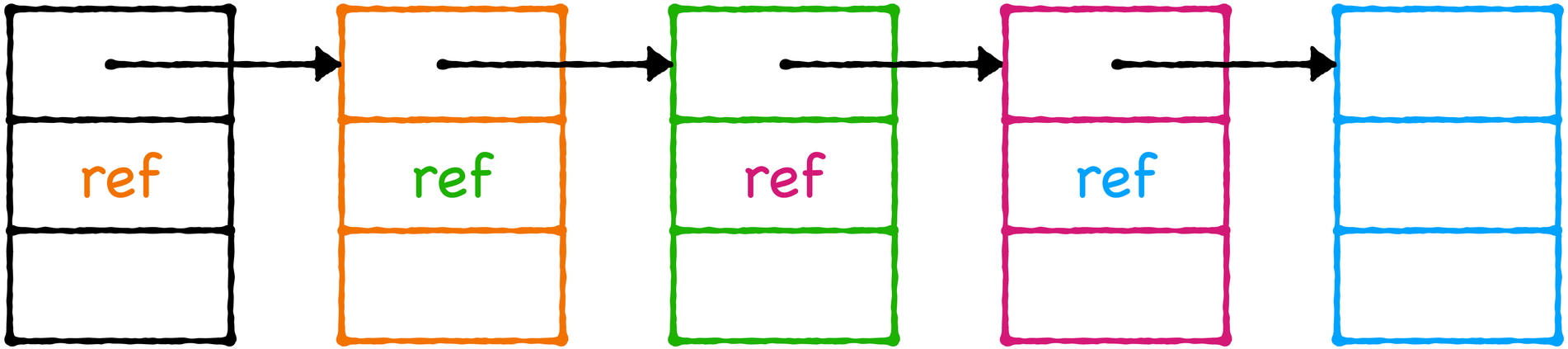

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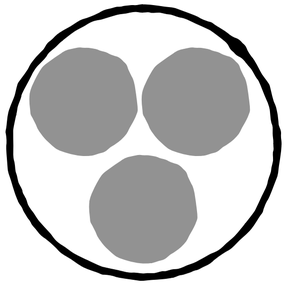

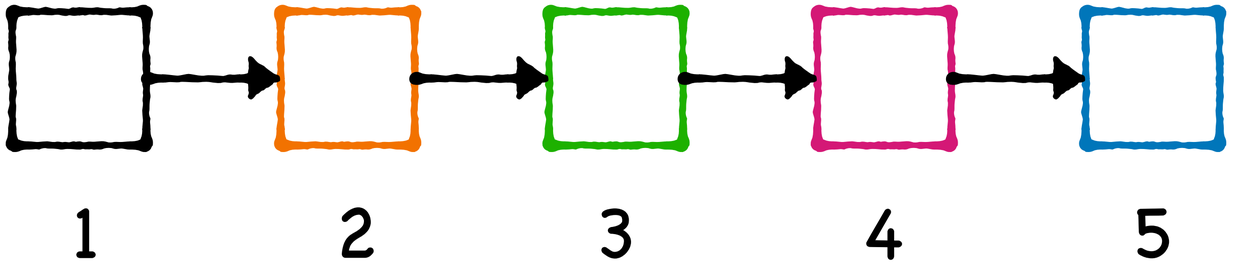

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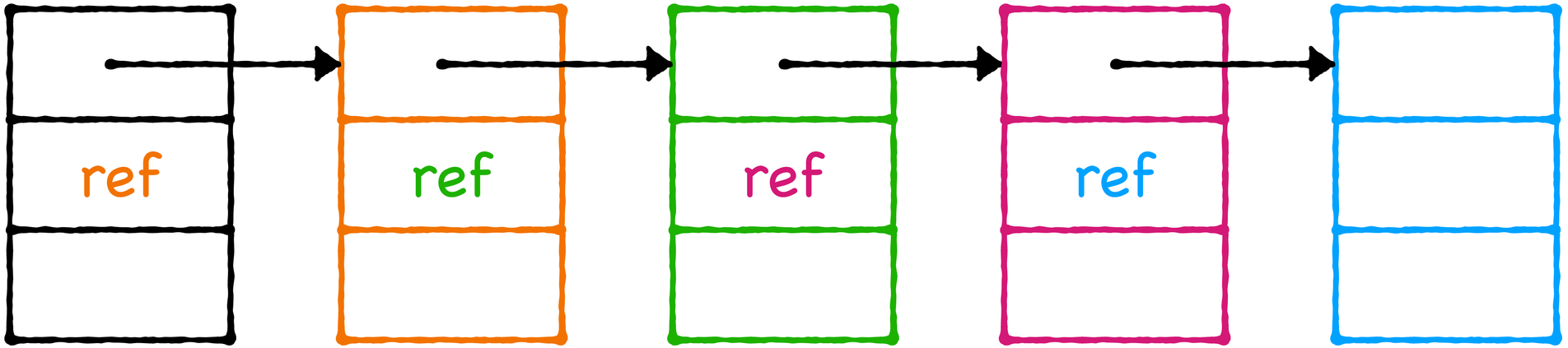

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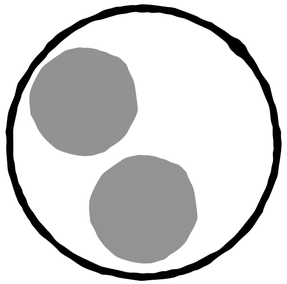

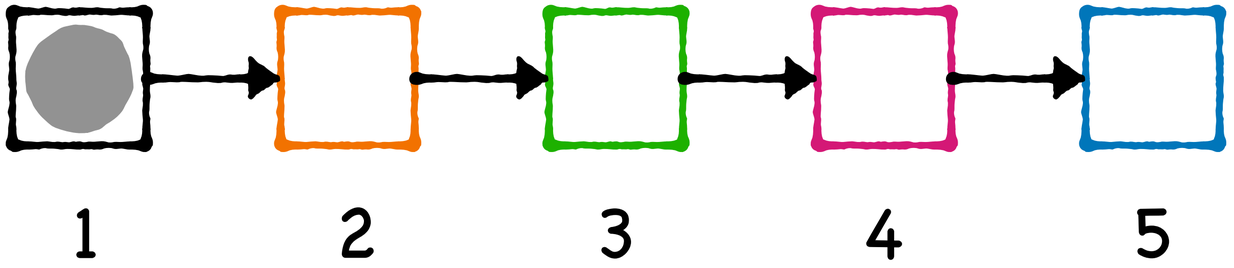

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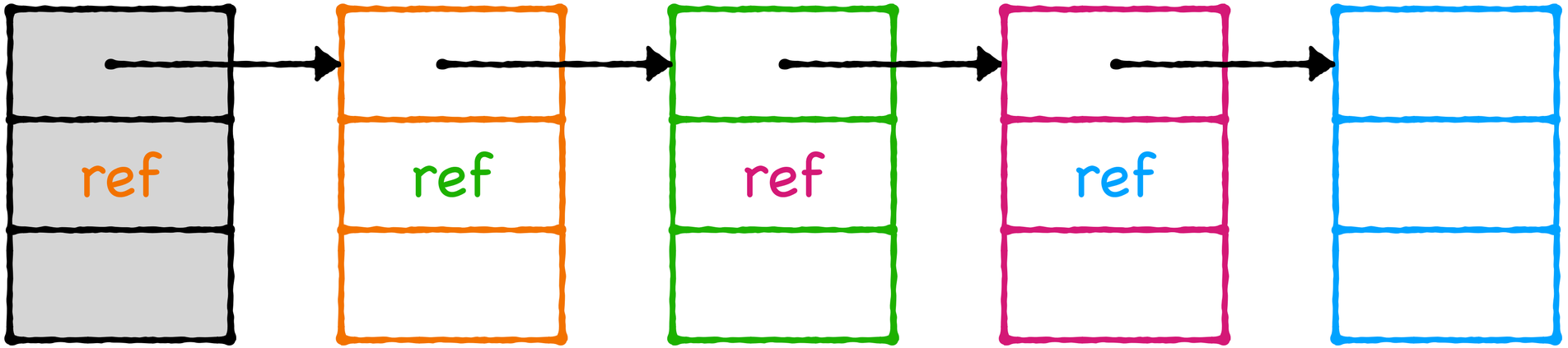

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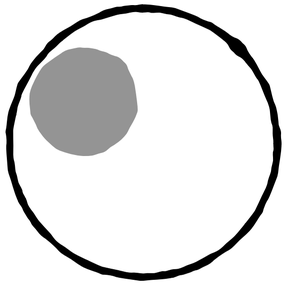

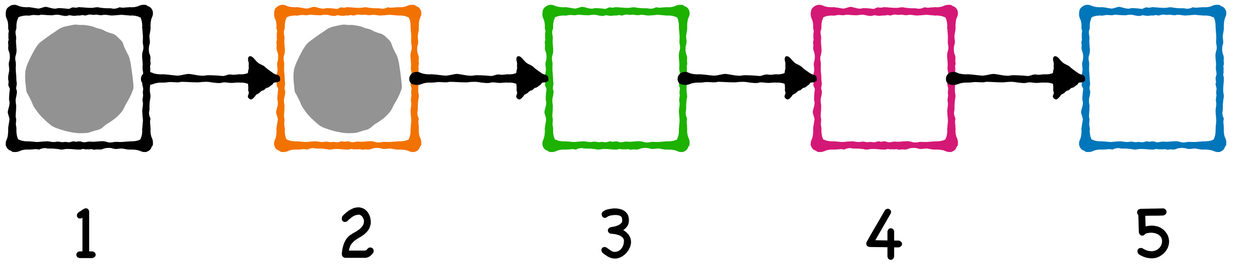

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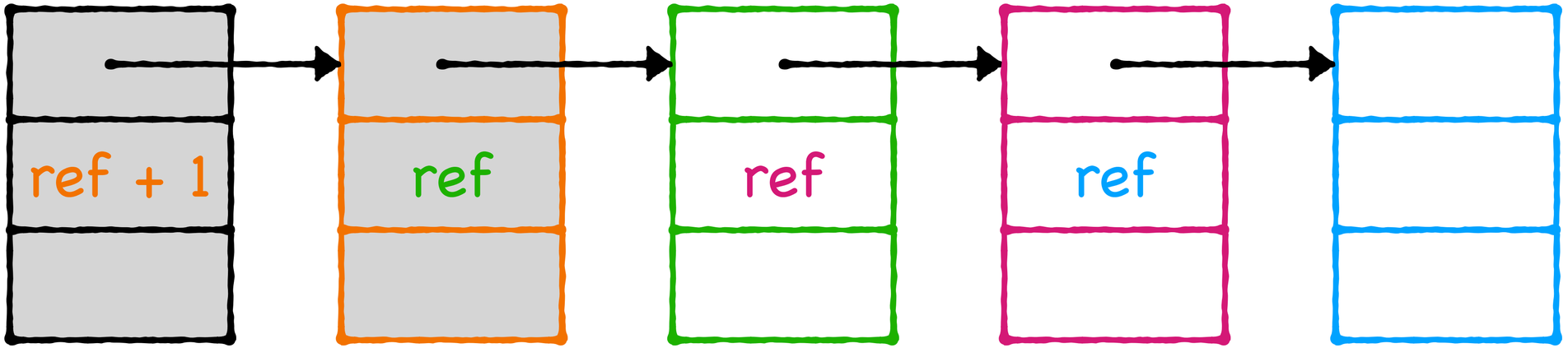

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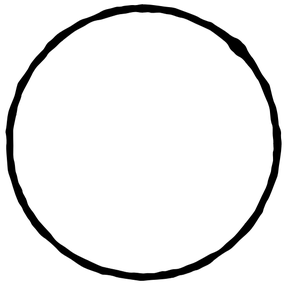

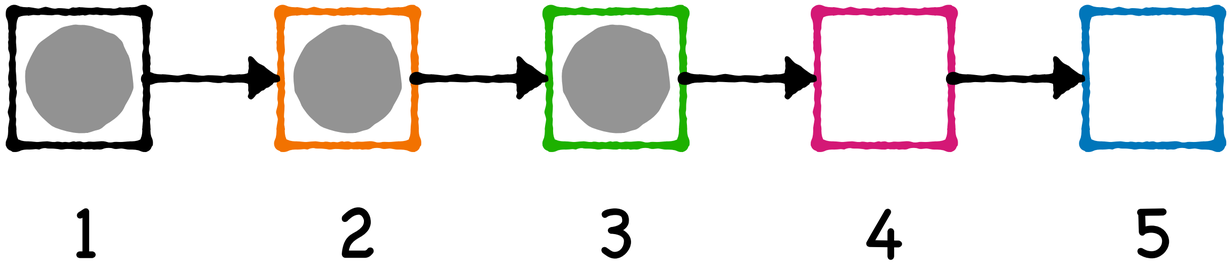

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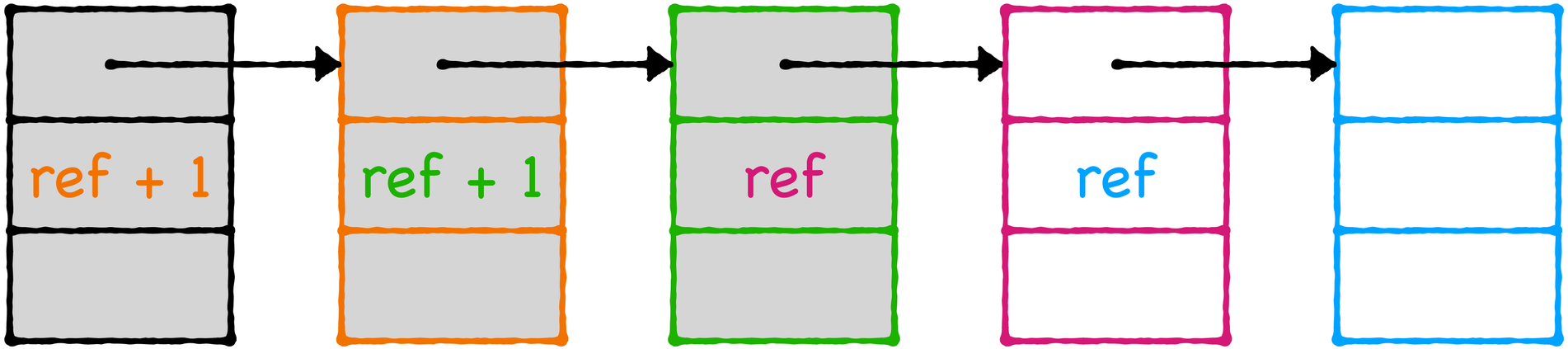

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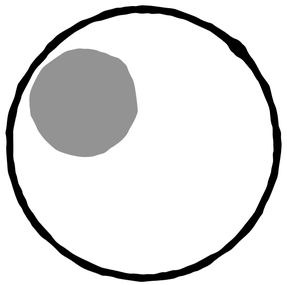

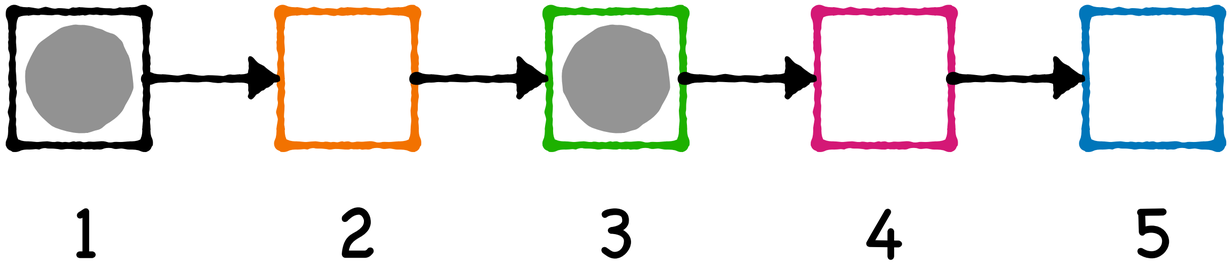

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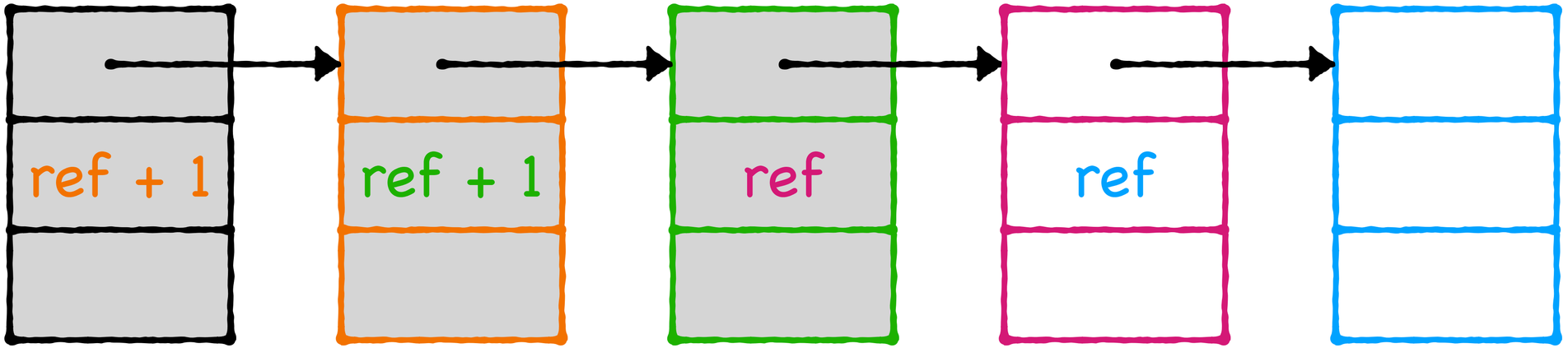

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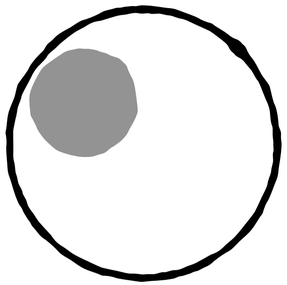

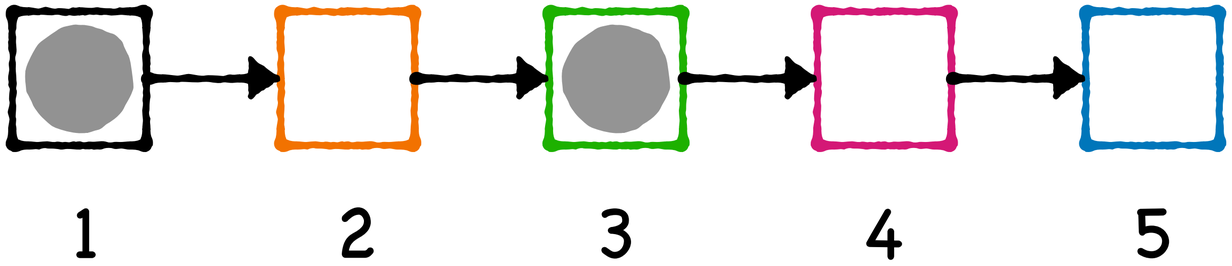

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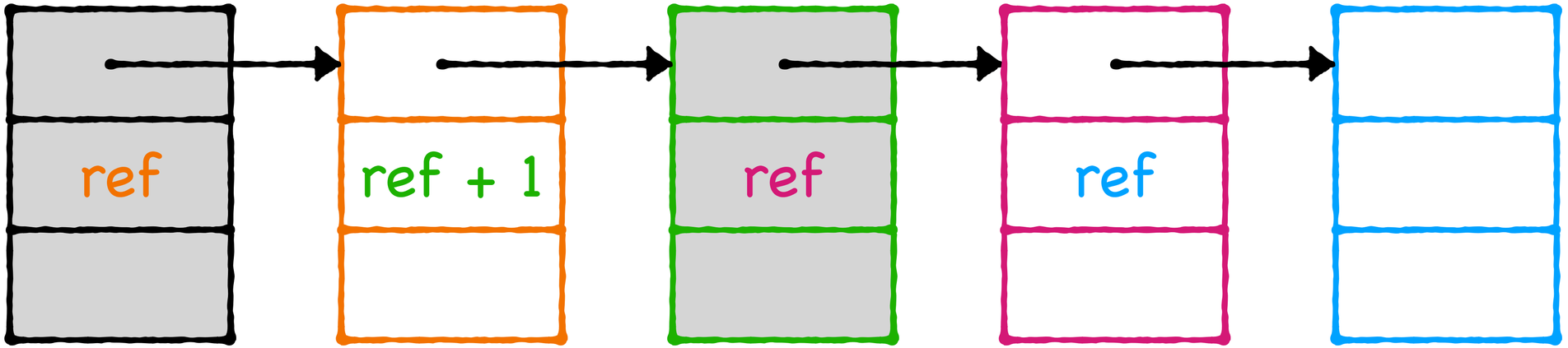

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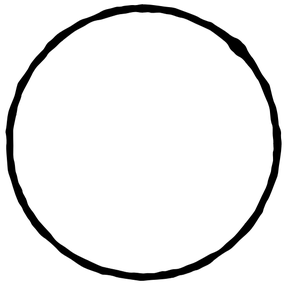

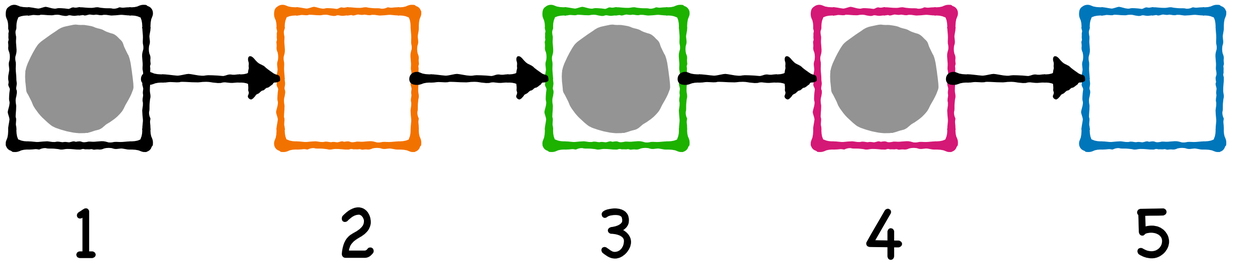

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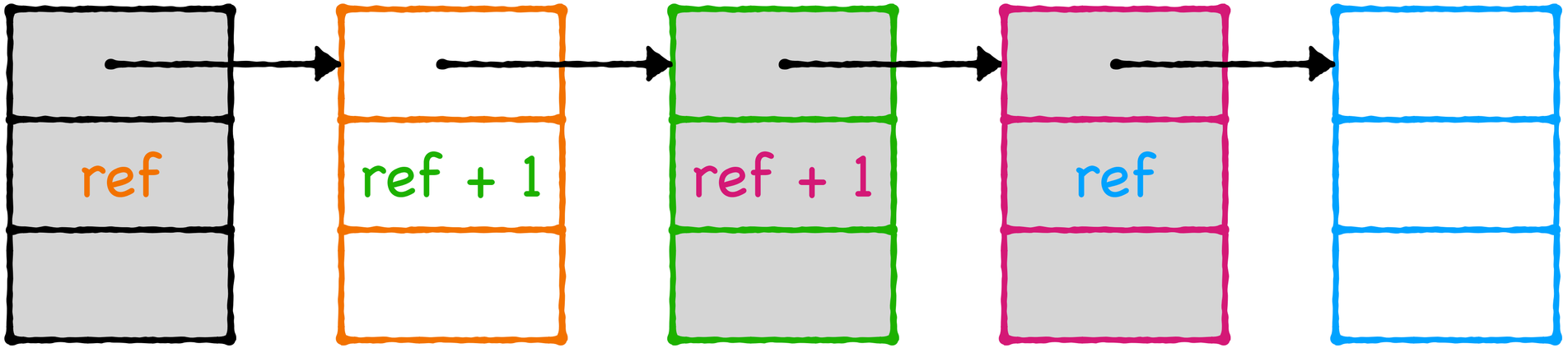

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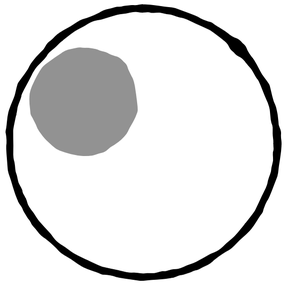

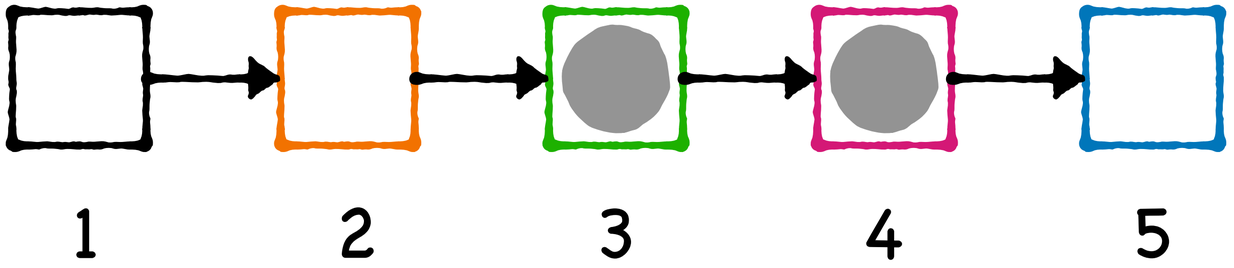

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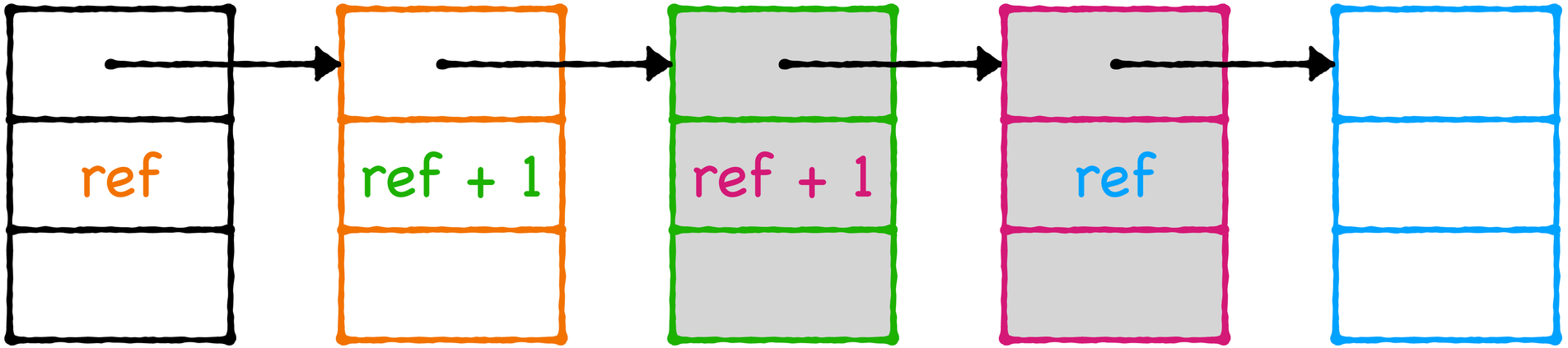

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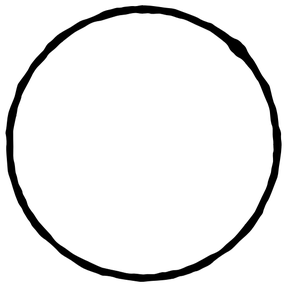

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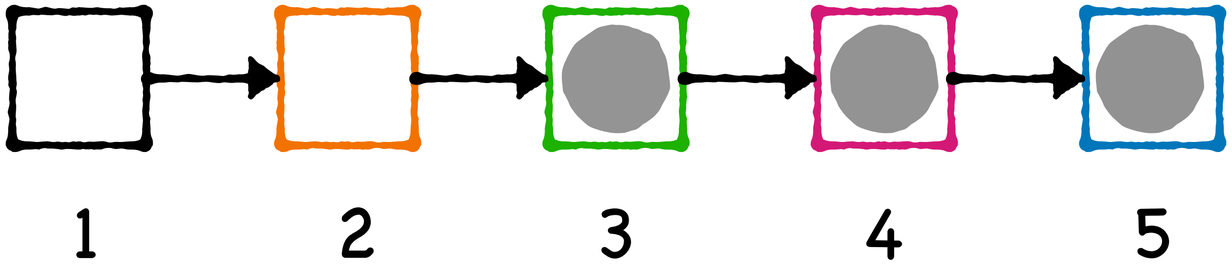

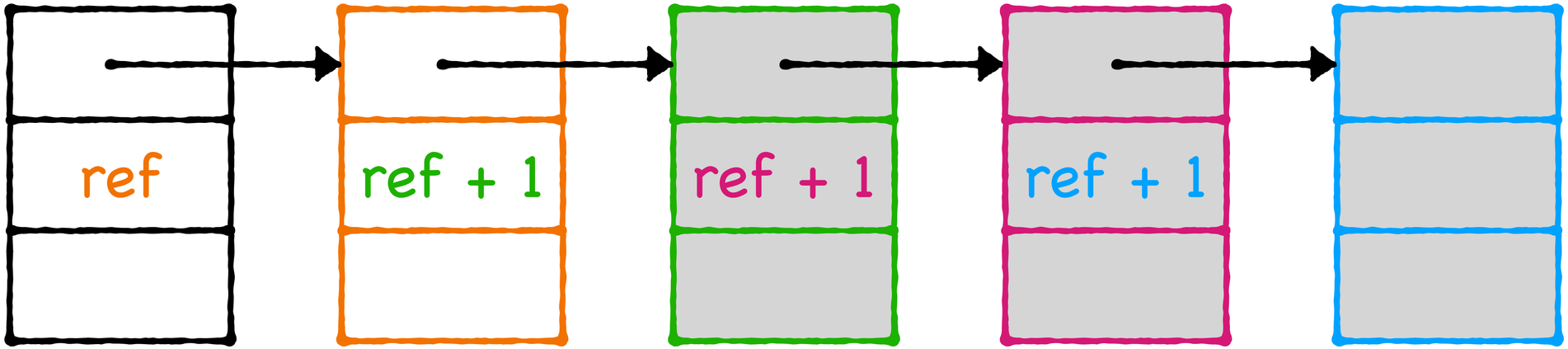

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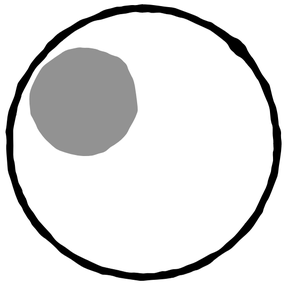

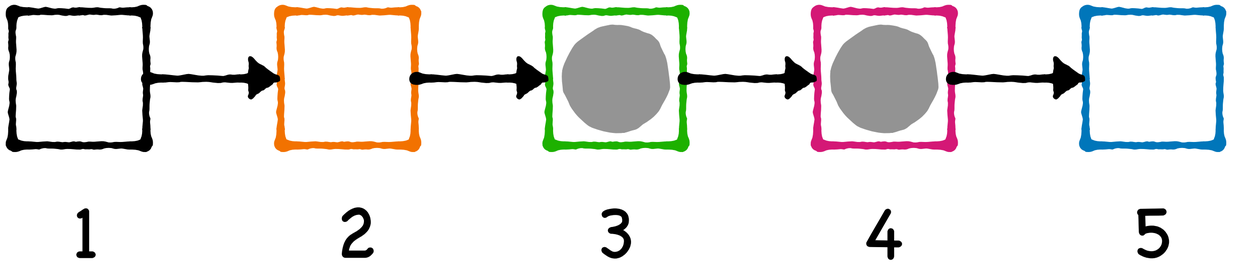

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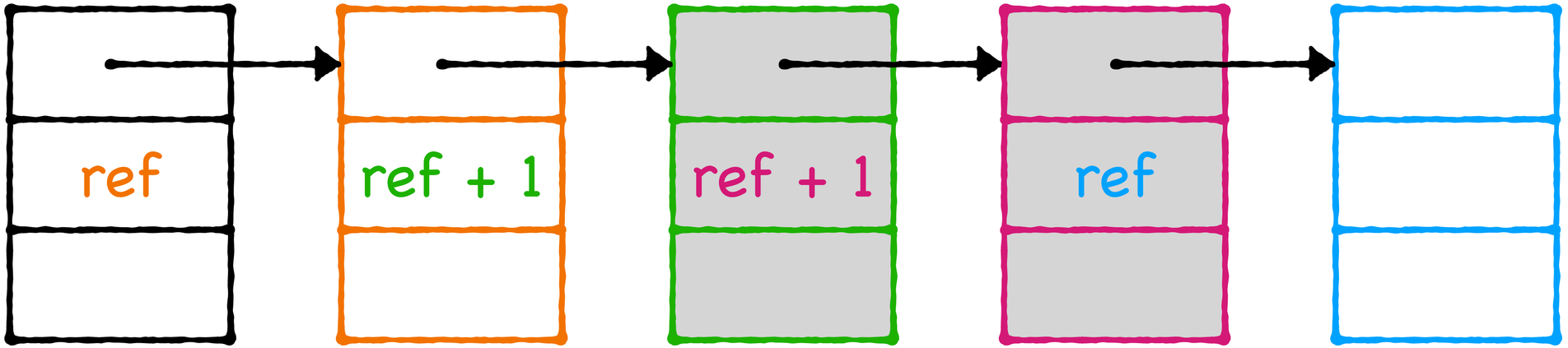

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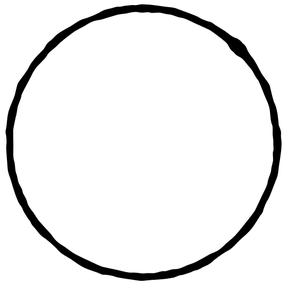
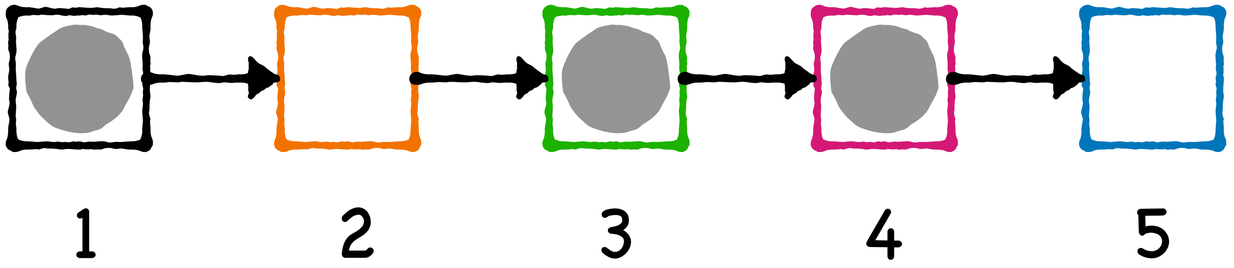
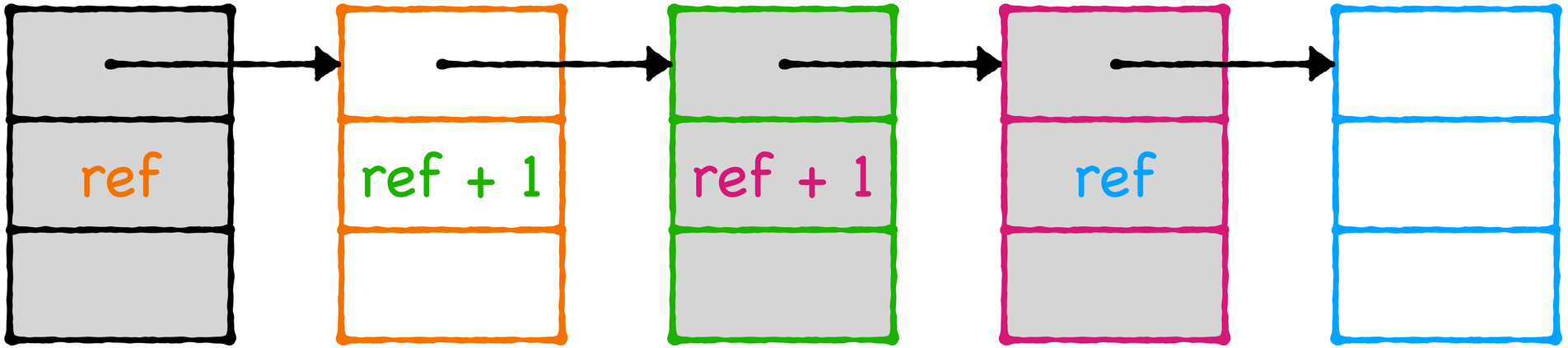
# of acquired cells: 2



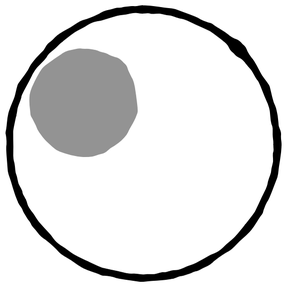


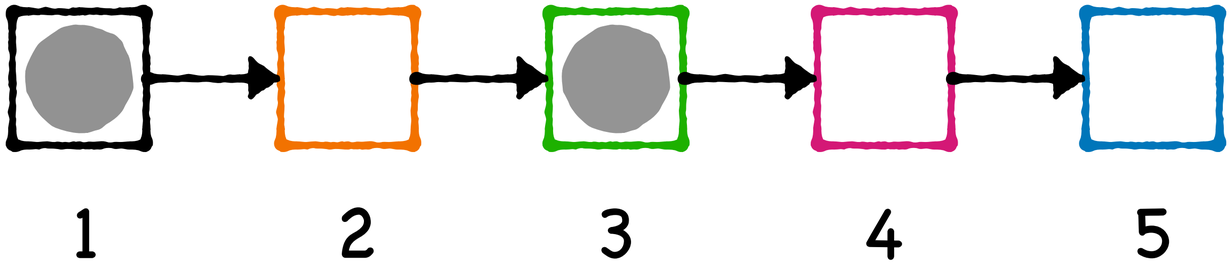
Pebbles



Head

# of acquired cells: 3





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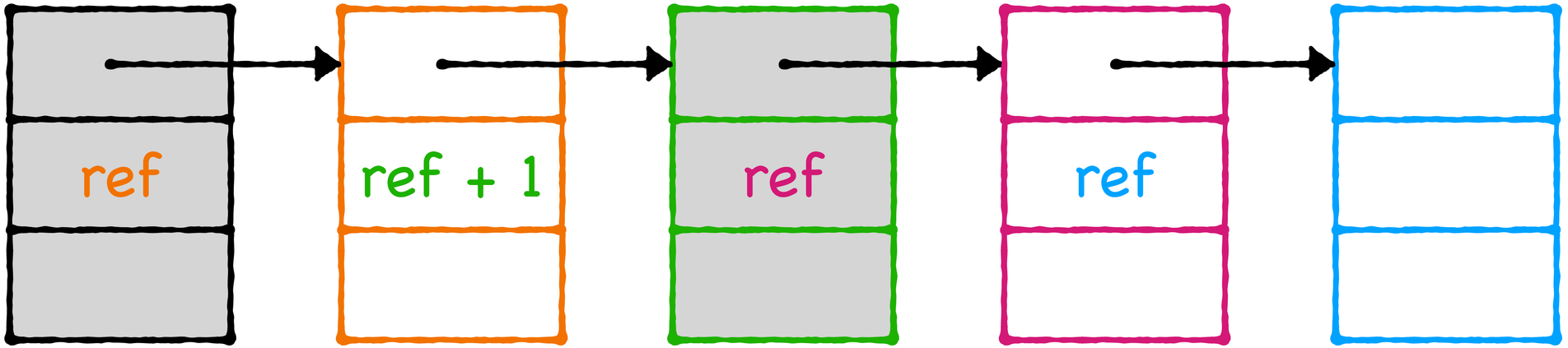

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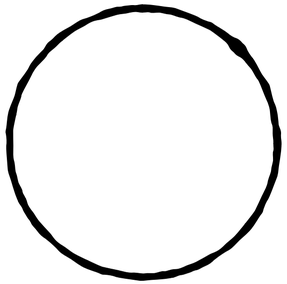

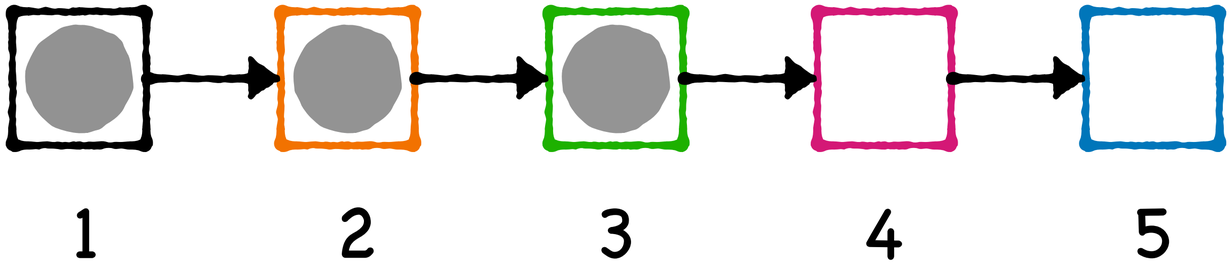

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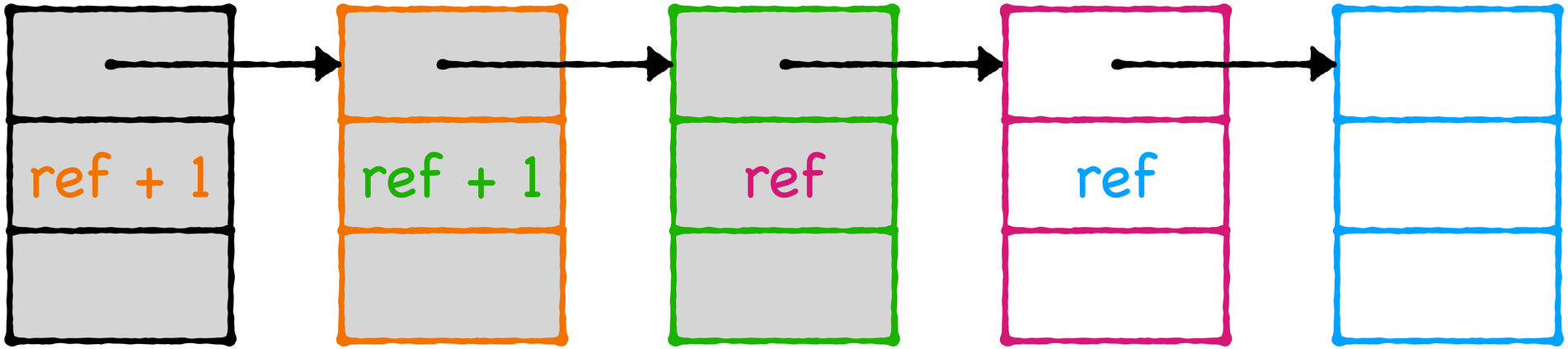

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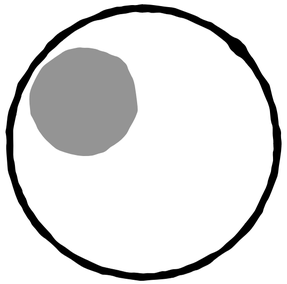

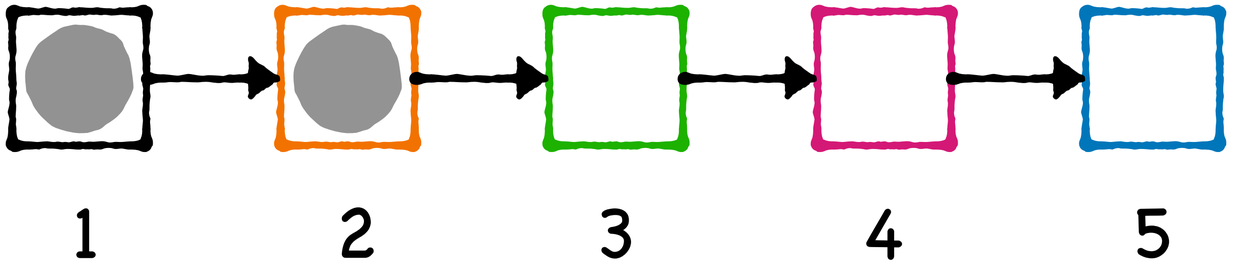

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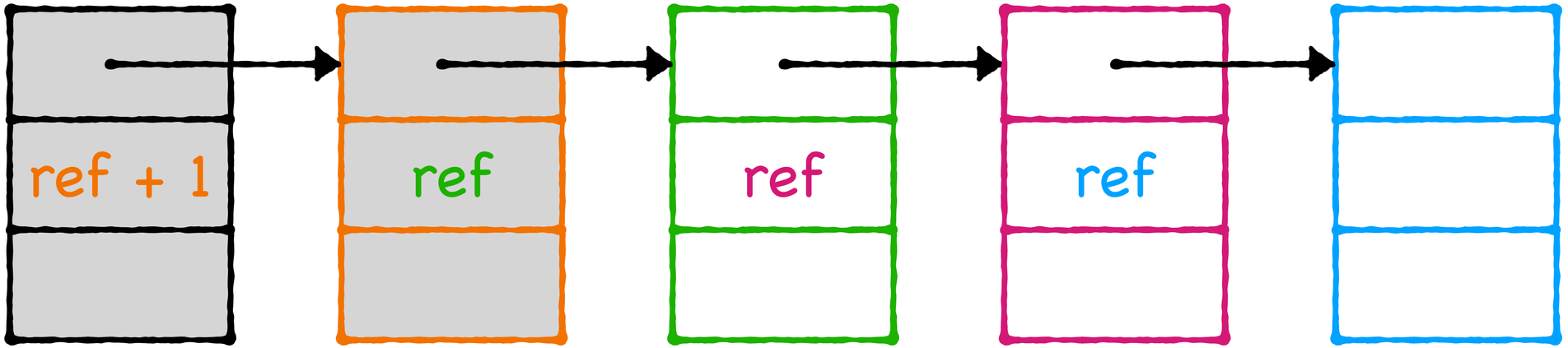

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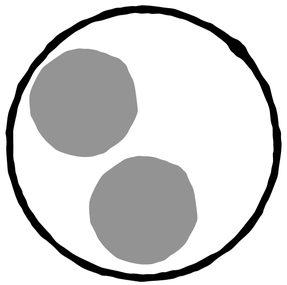

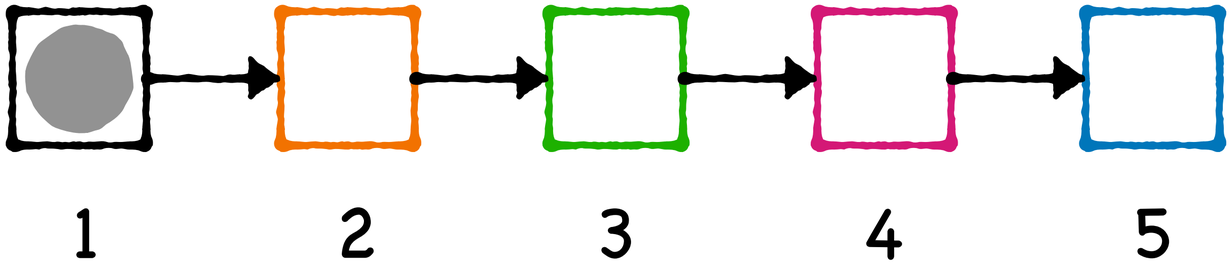

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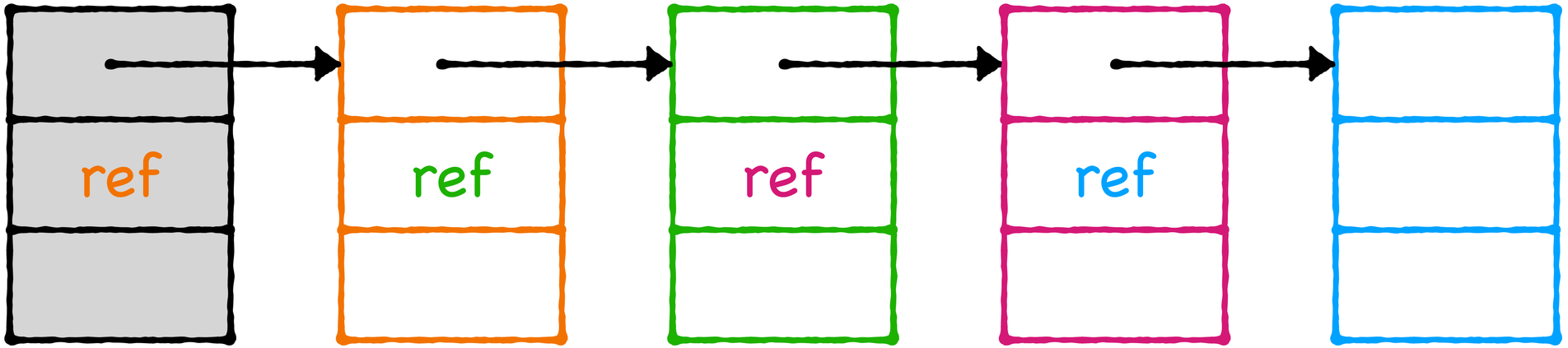

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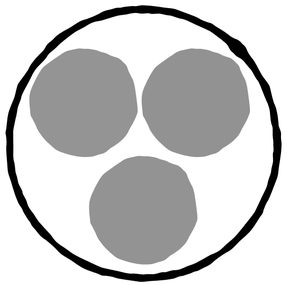
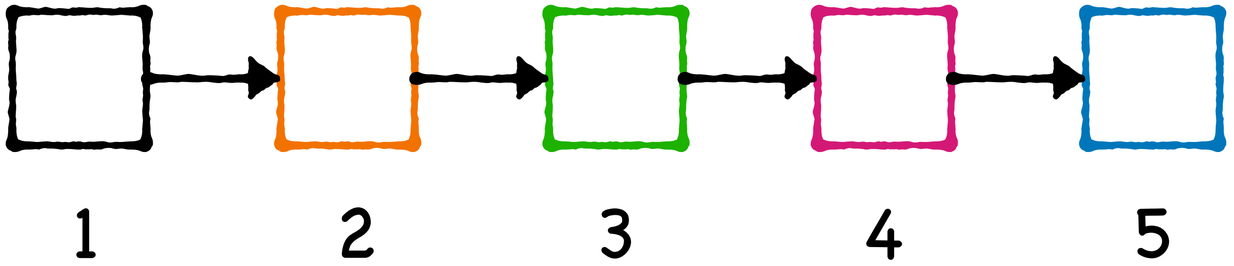
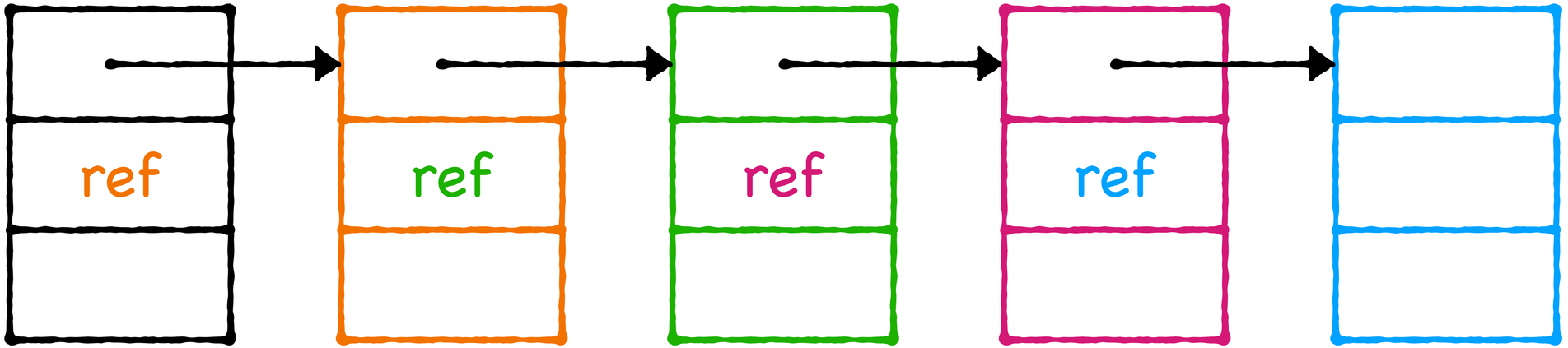




Pebbles



Head

# of acquired cells: 0

Lower Bound on the Number of Pebbles Needed

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Theorem[Li&Vitanyi'97]: You can win the game for a list of n nodes with m pebbles **only if** $m > \log_2(n)$

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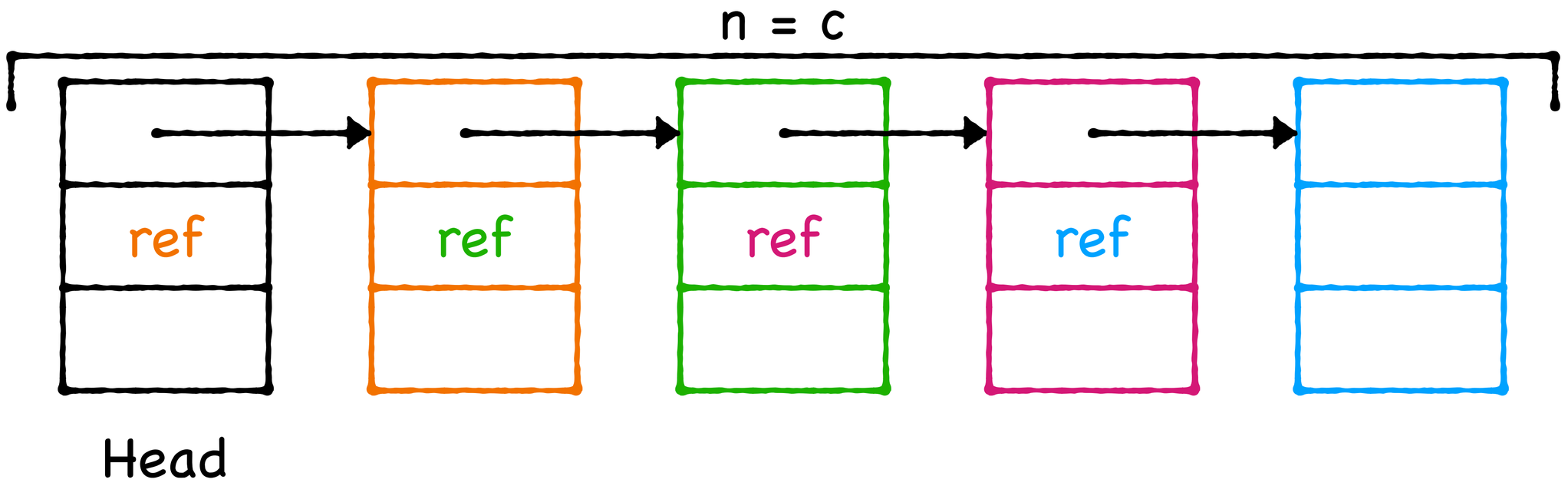

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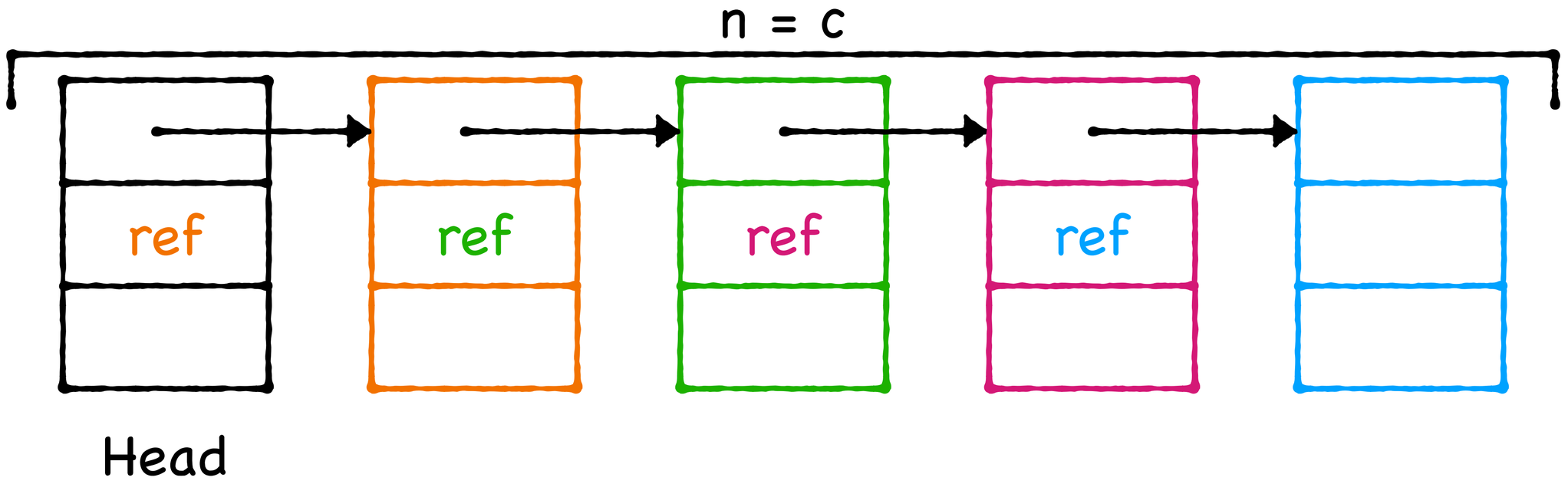
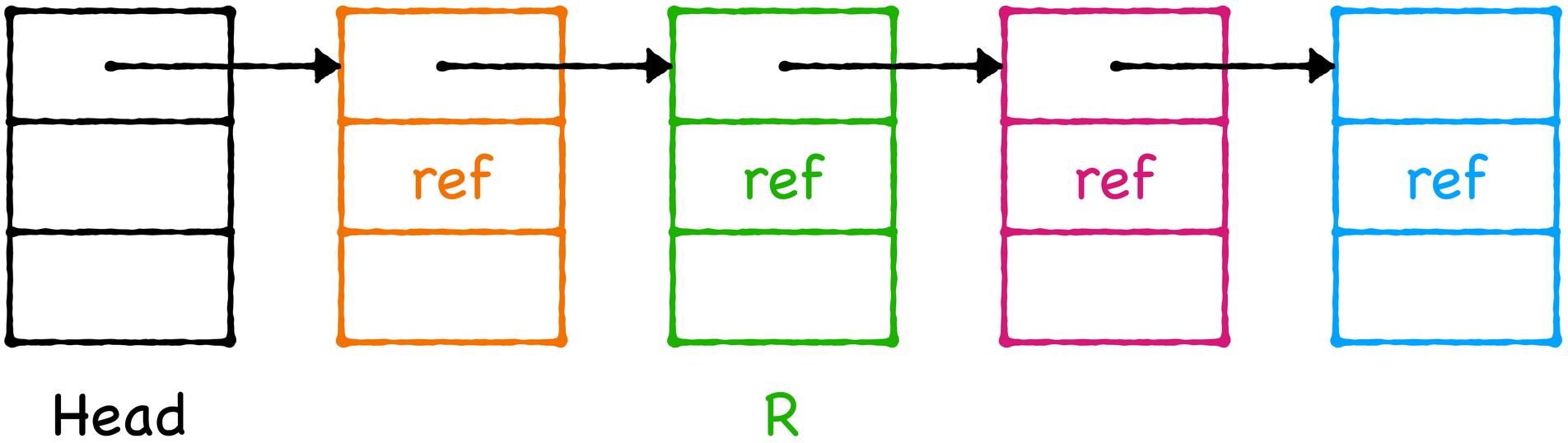


The space complexity is $\Omega(c\log(c))$ where c is the point contention

This is not good enough:
We want O(c) not O(clog(c))!

- Split each reference counter into an acquisition and revocation counter



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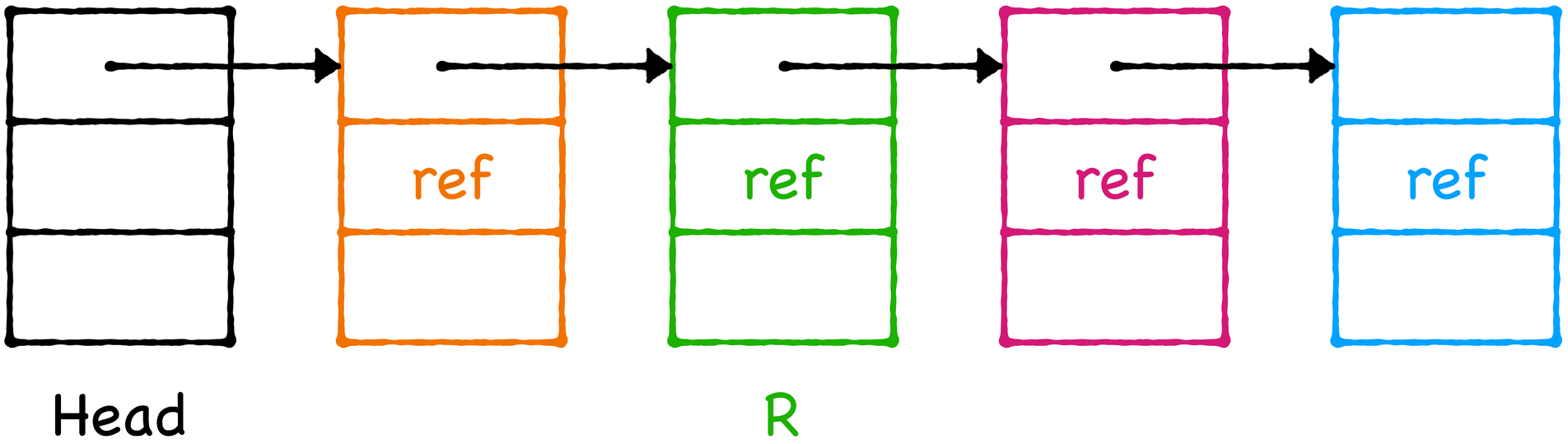

- acquisition: # of processes that have gained the right to access a cell

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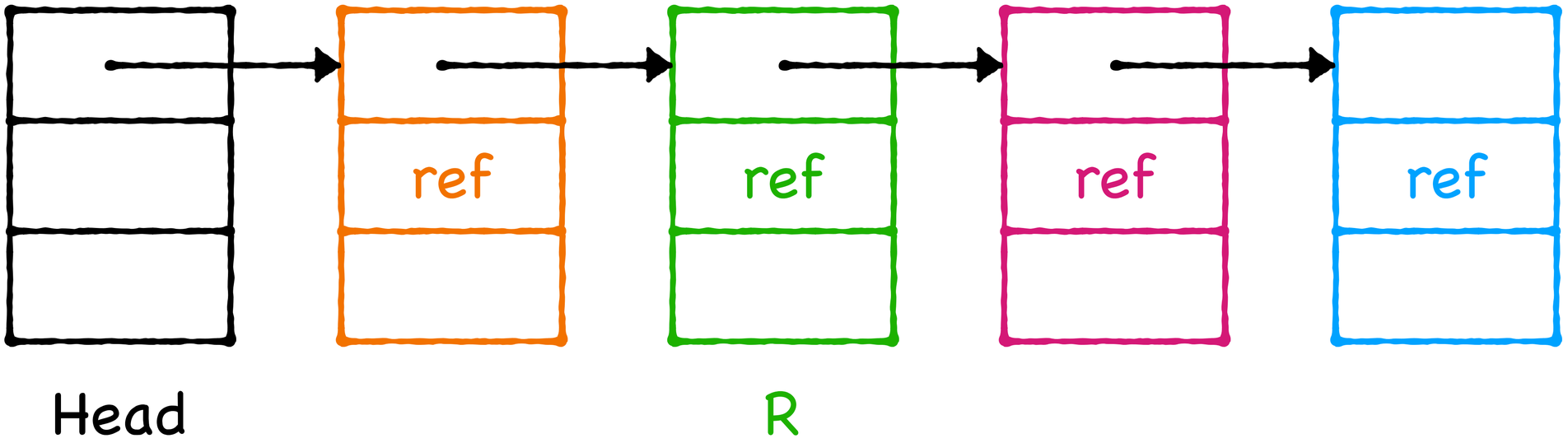
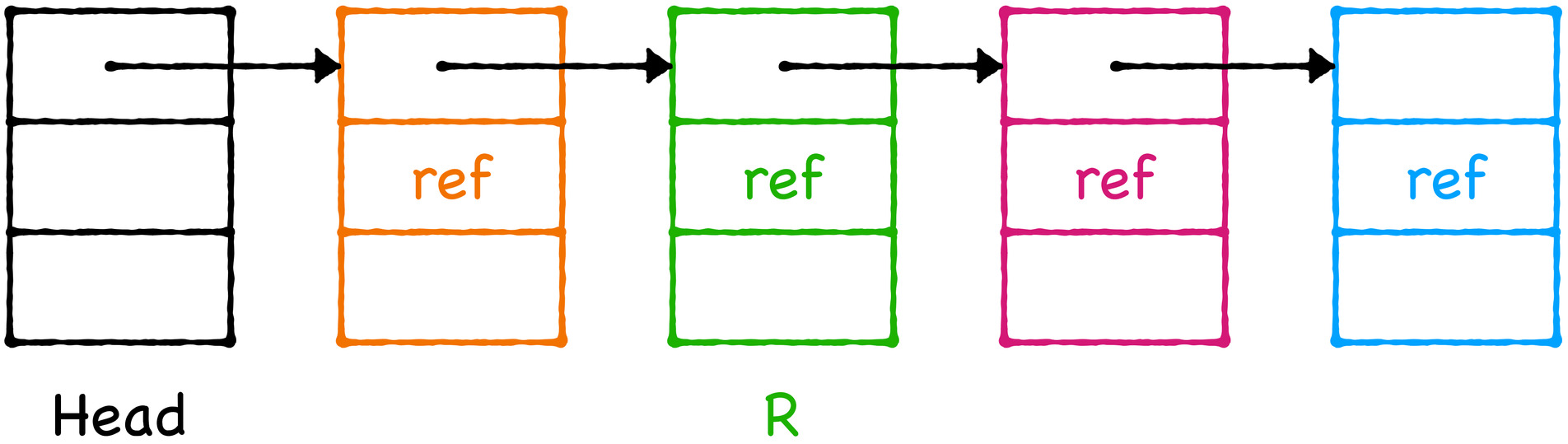


- acquisition: # of processes that have gained the right to access a cell
- revocation: # of processes that have relinquished that right

- Split each reference counter into an acquisition and revocation counter

ref = acquisition - revocation



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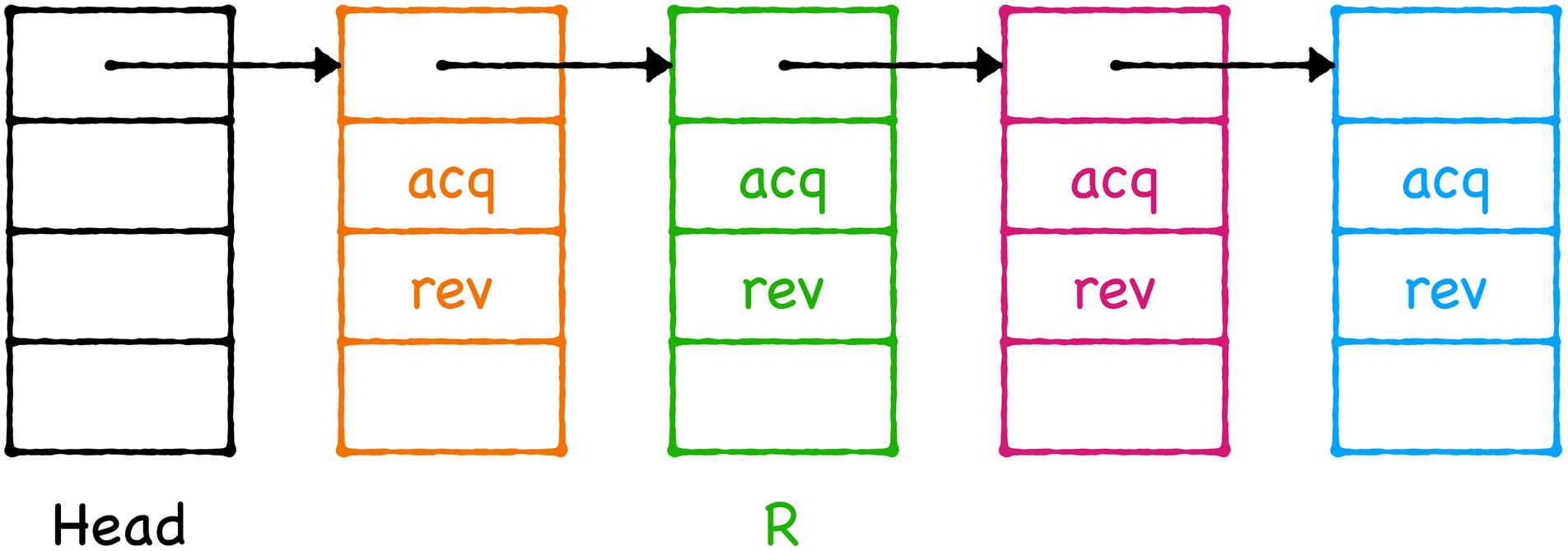

- Split each reference counter into an acquisition and revocation counter
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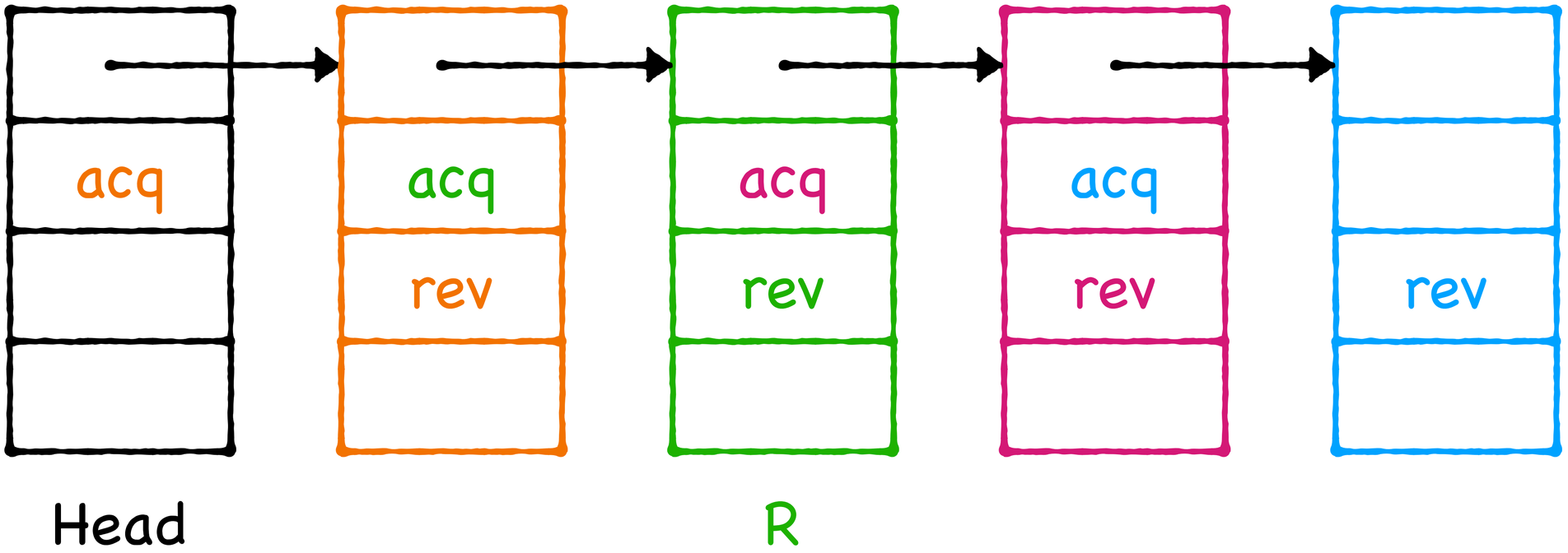

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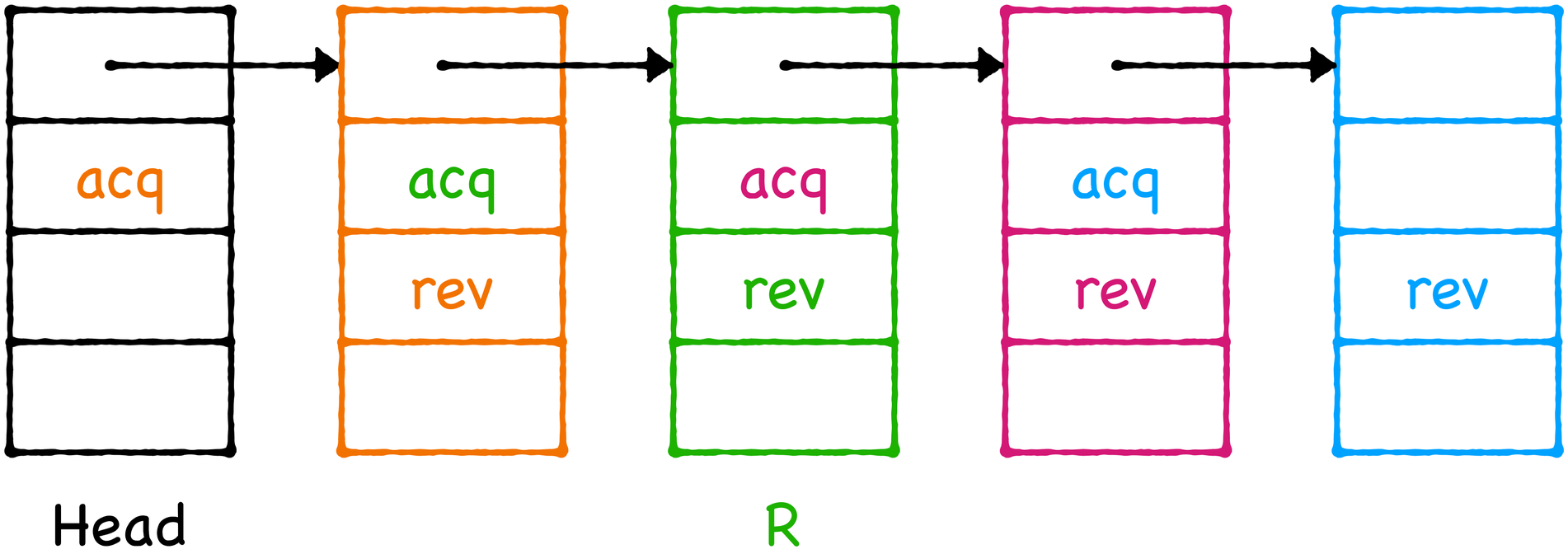
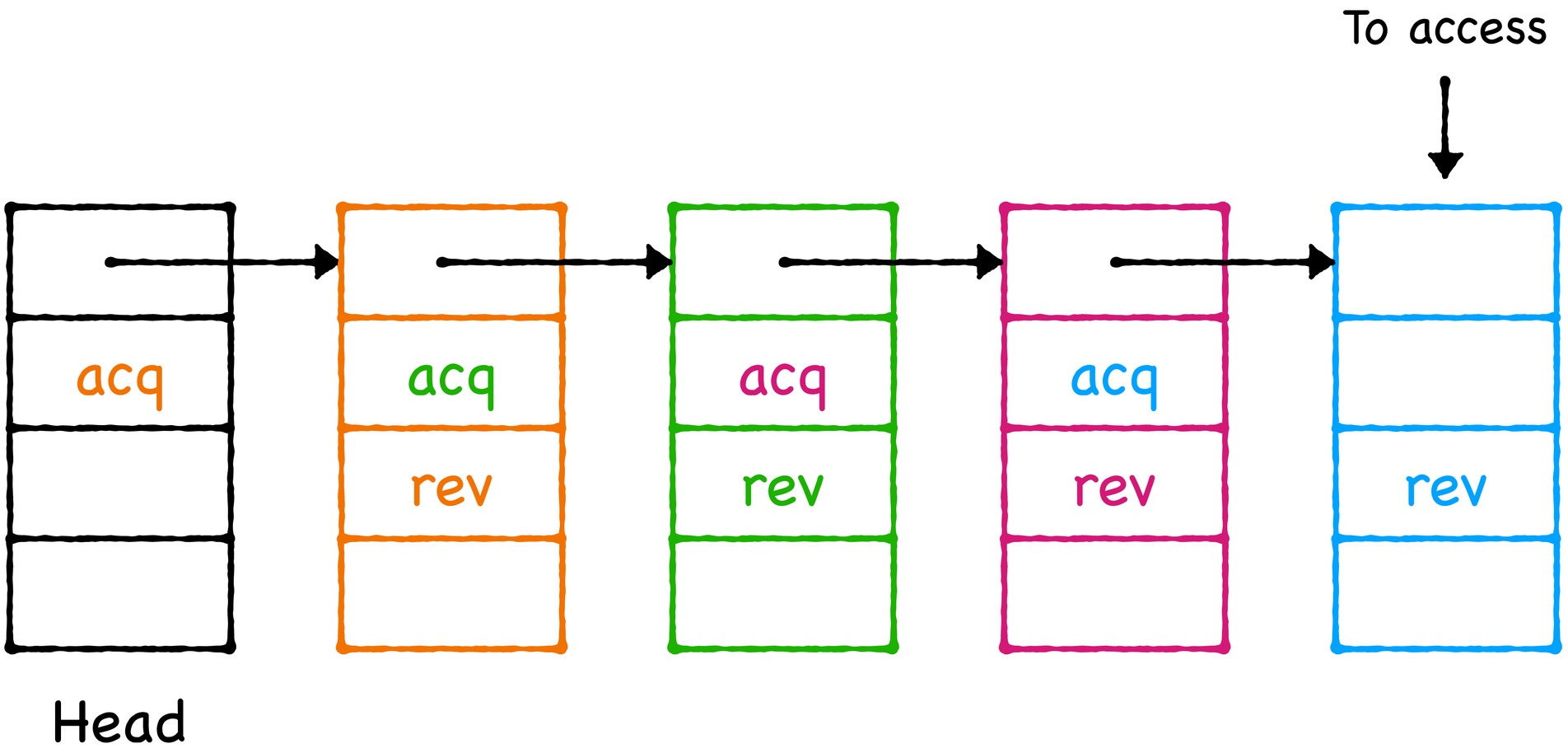
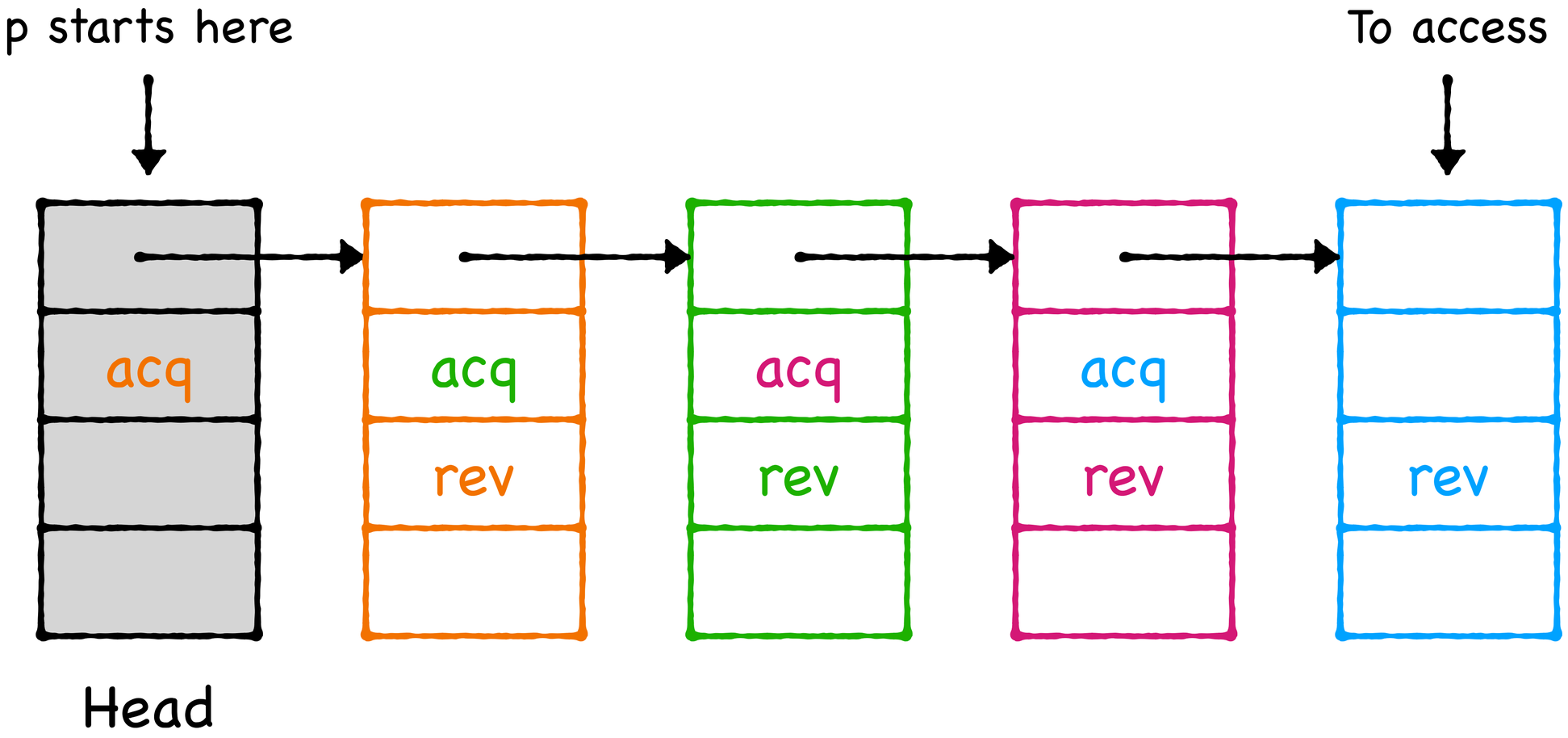
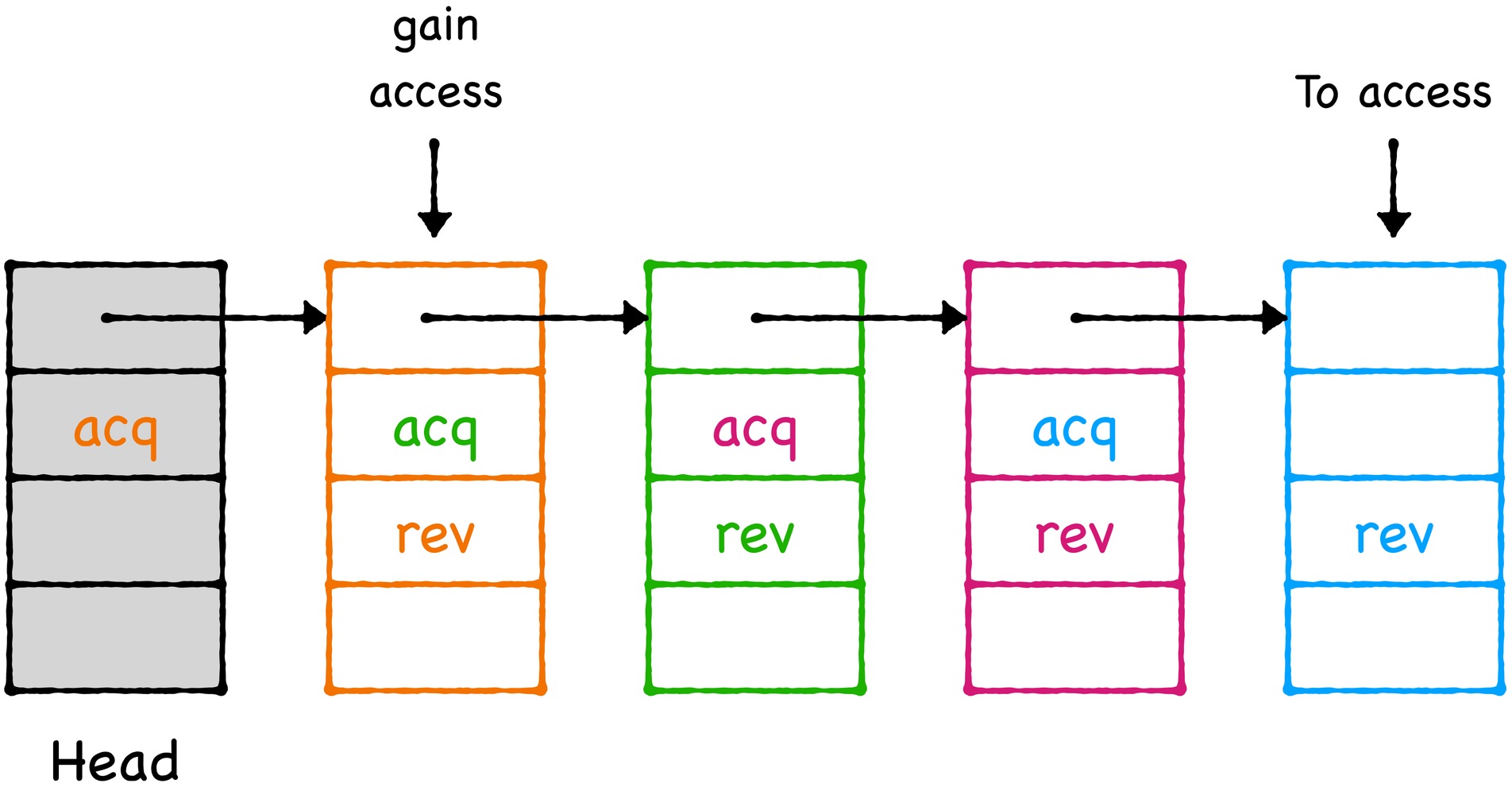


There is no chicken-and-egg issue for relinquishing access to cells!


To access
acq
acq
rev
acq
rev
acq
rev
rev
Head



# of acquired cells: 1



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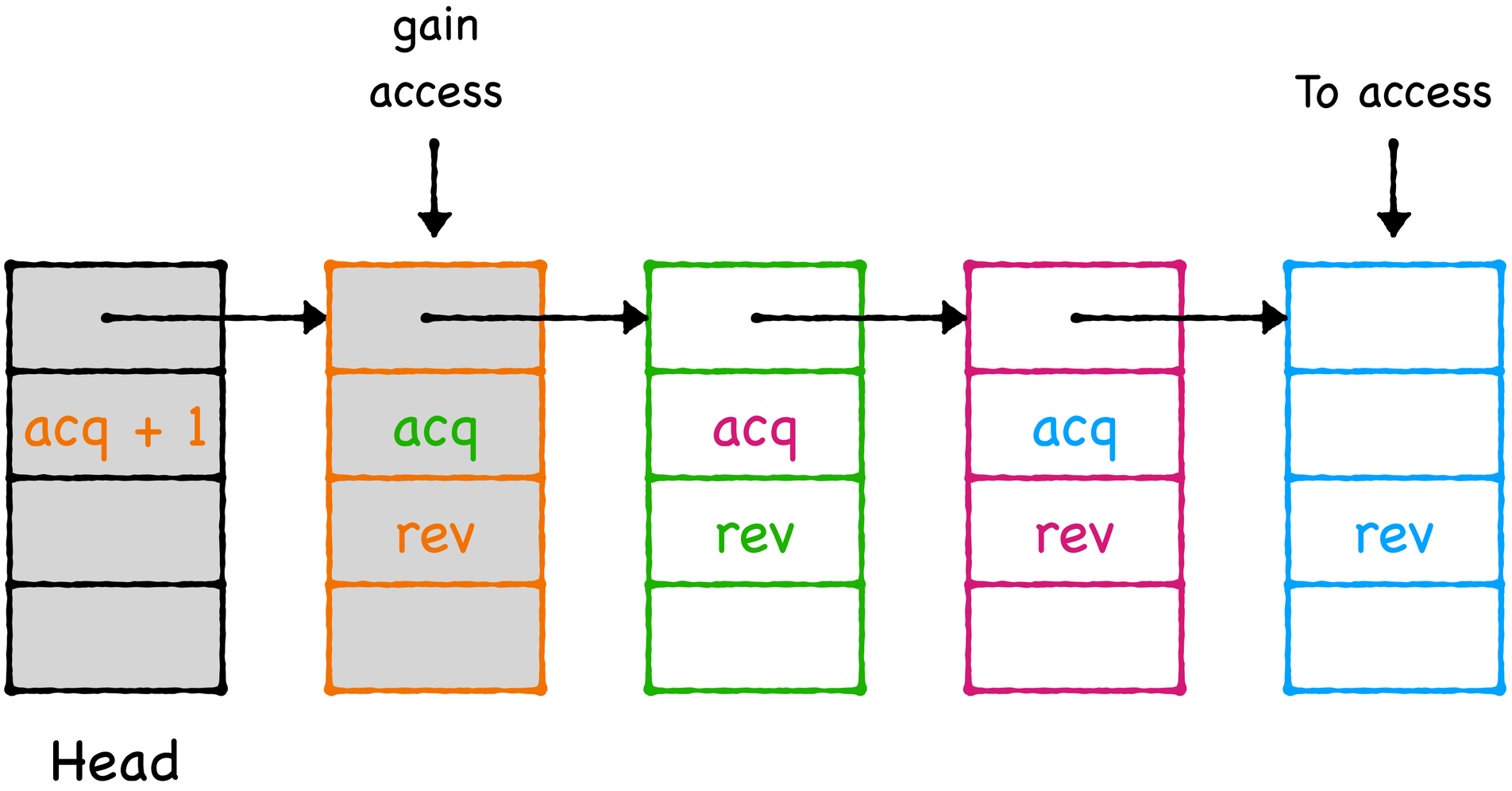
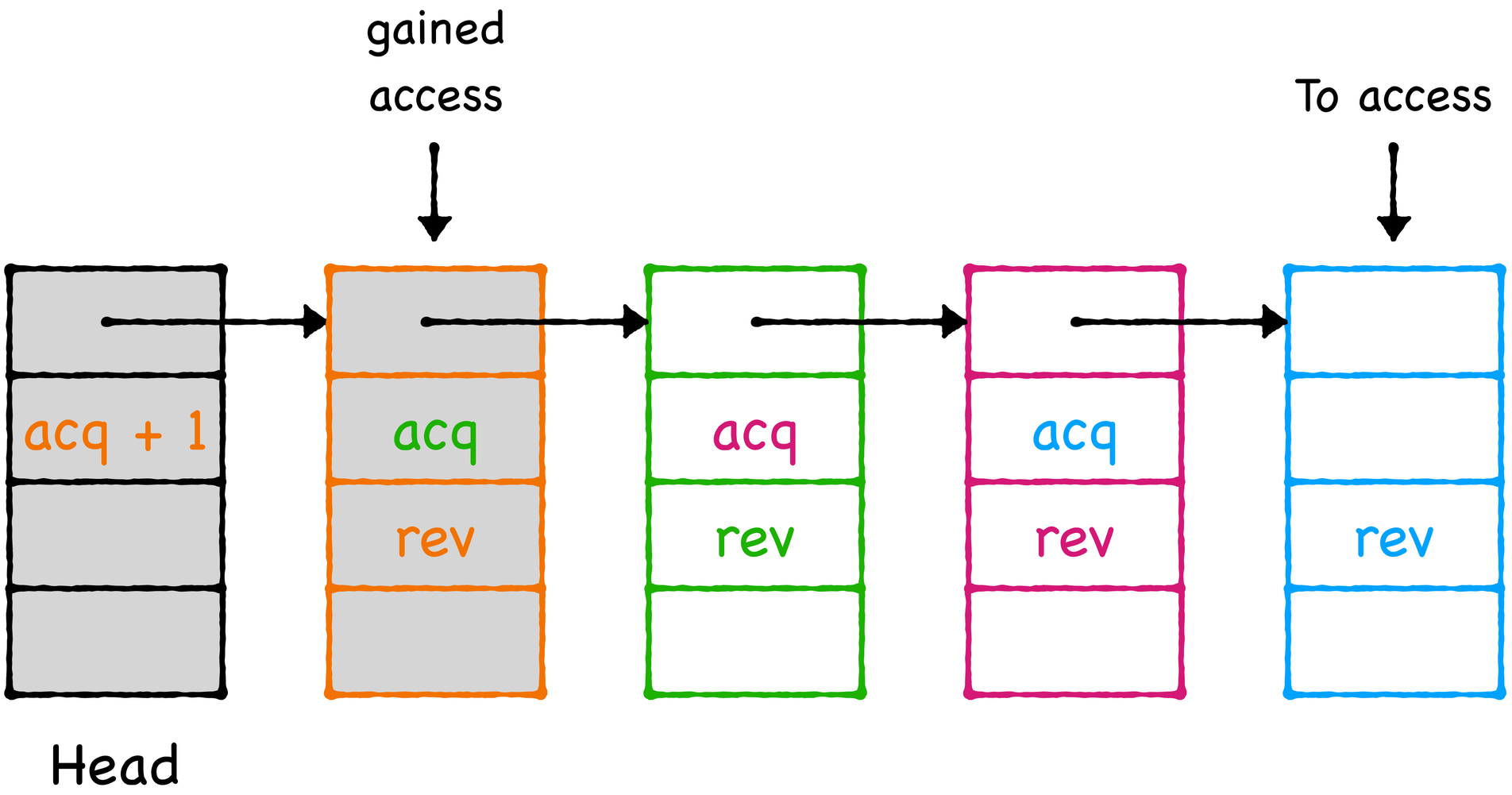


# of acquired cells: 2



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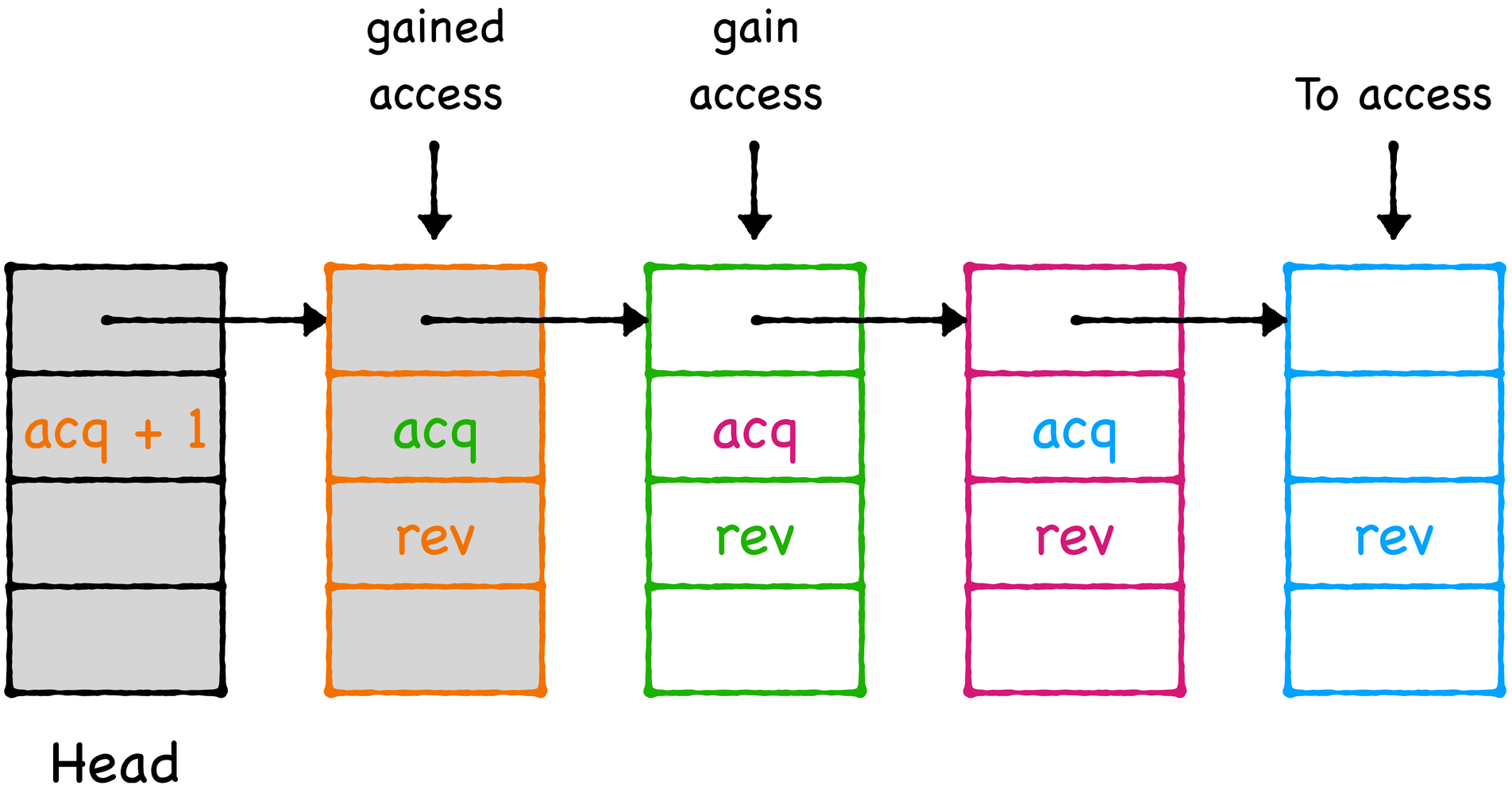

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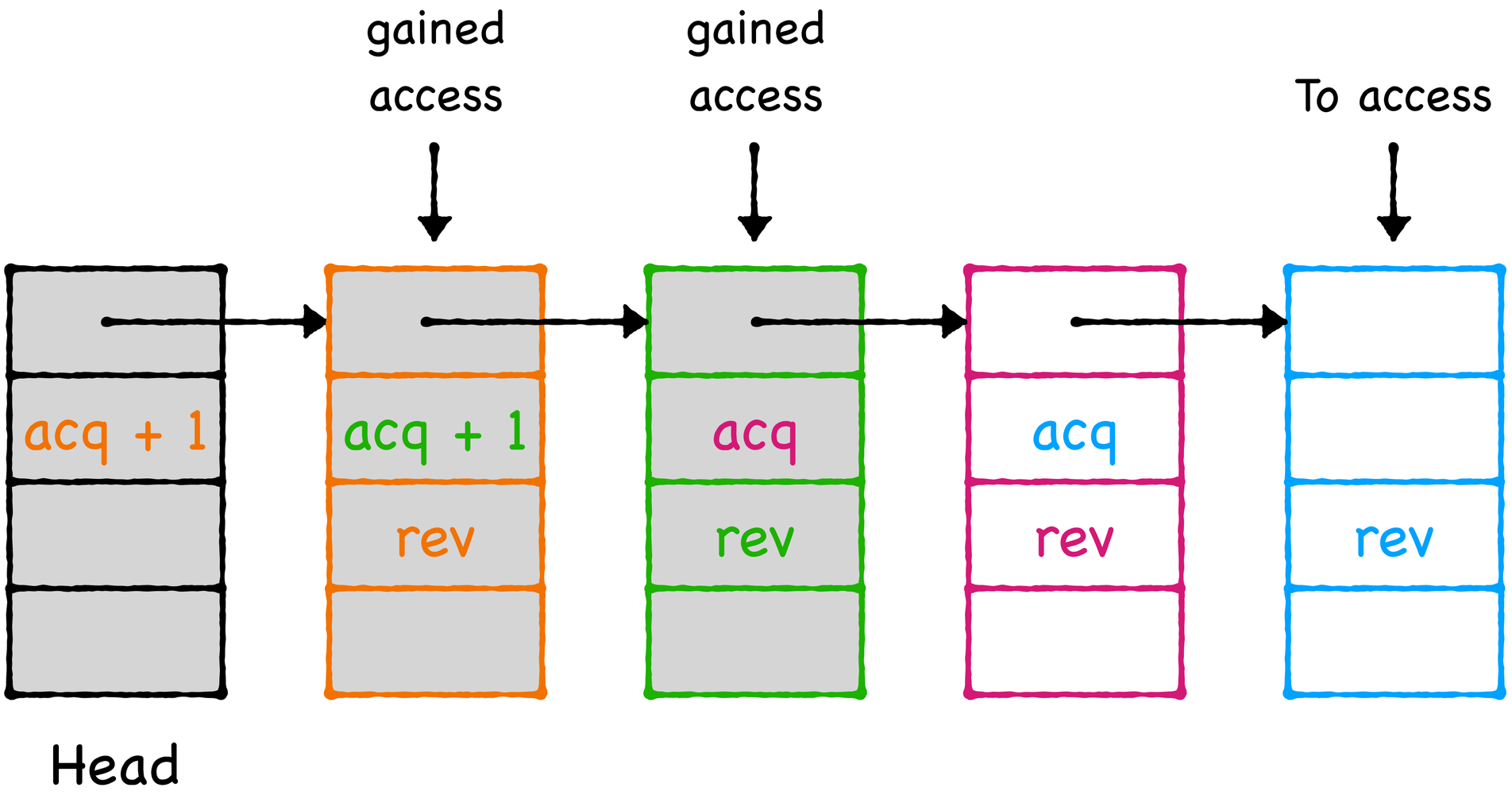
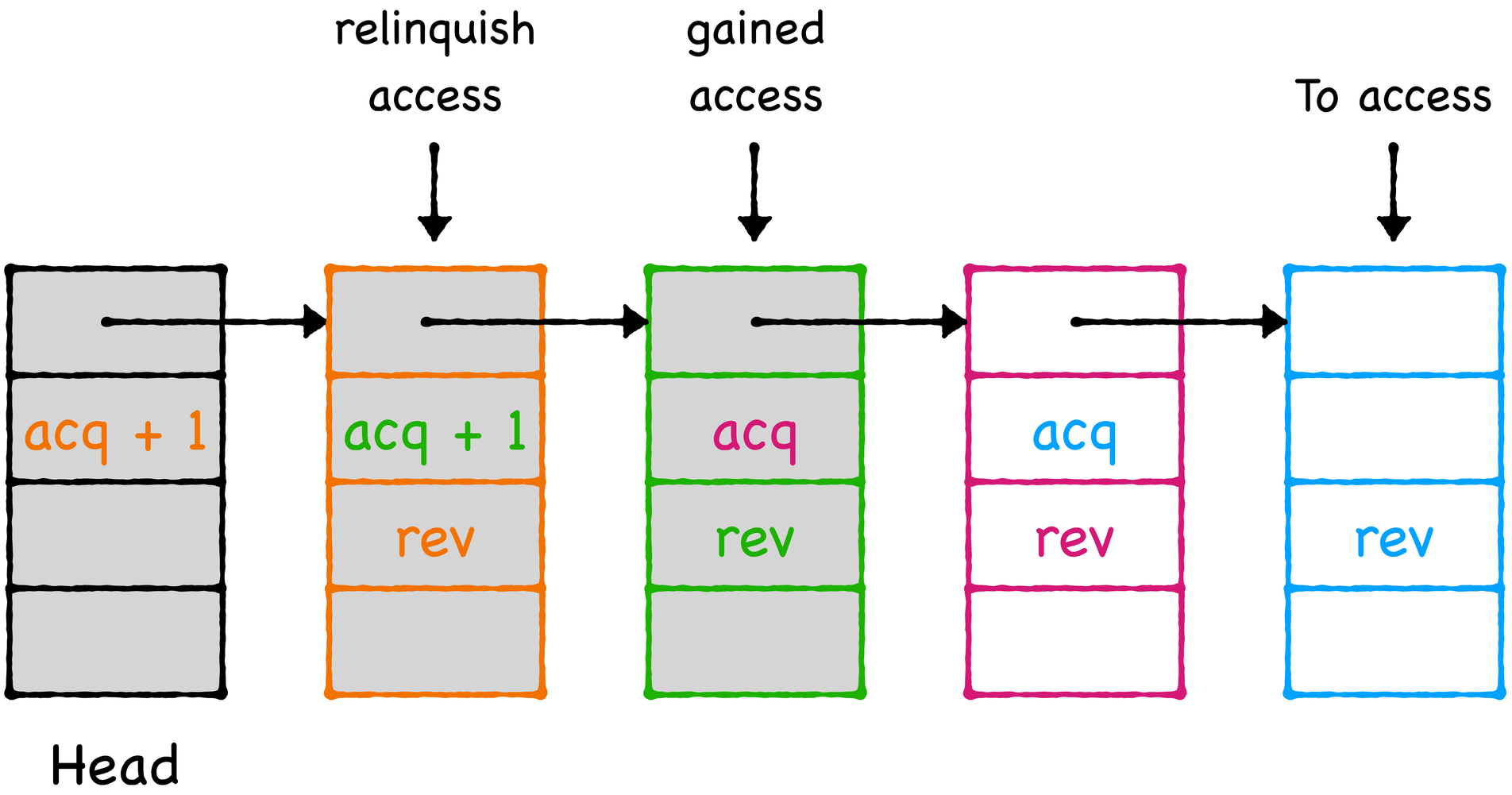


# of acquired cells: 3



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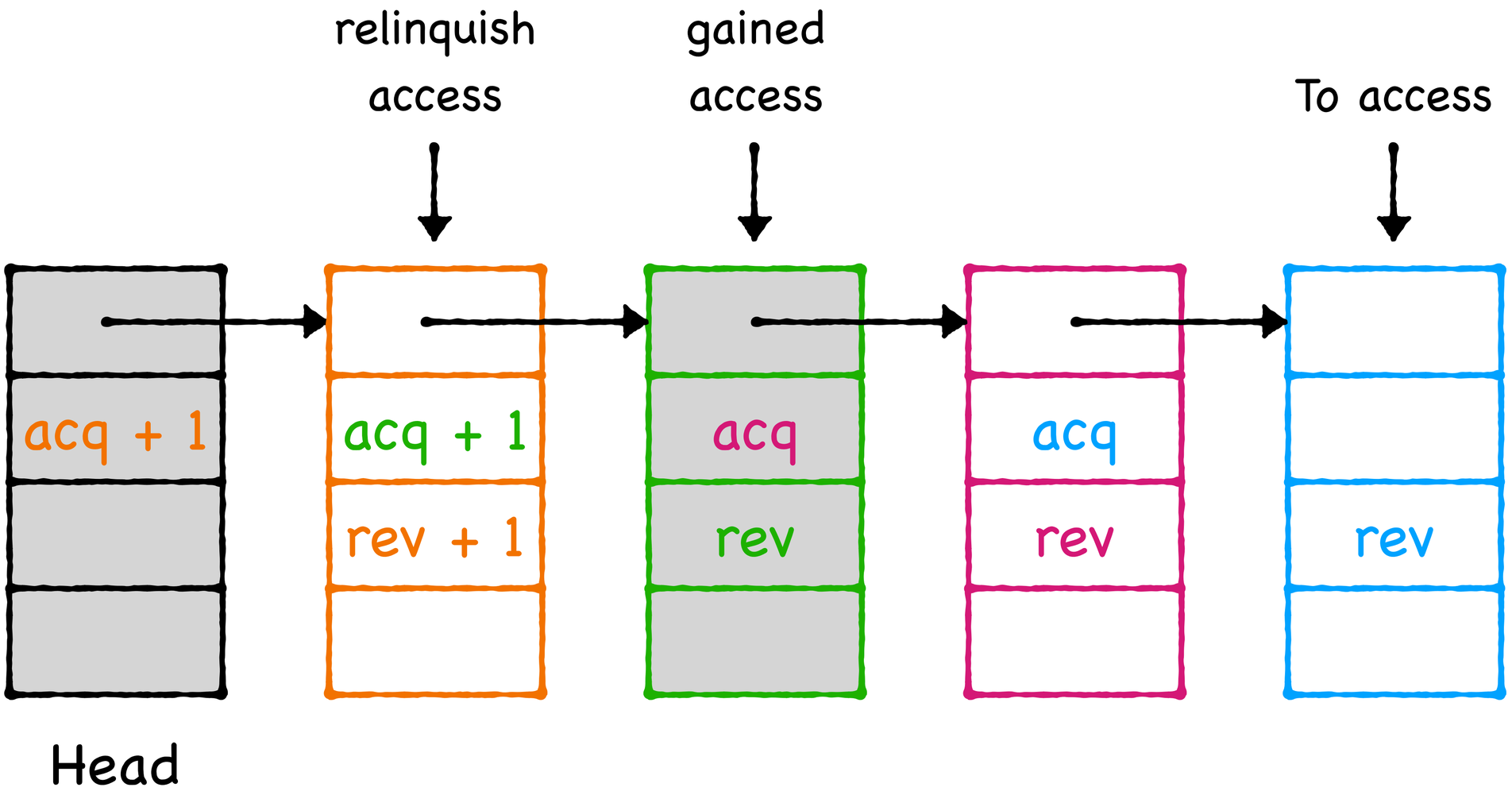

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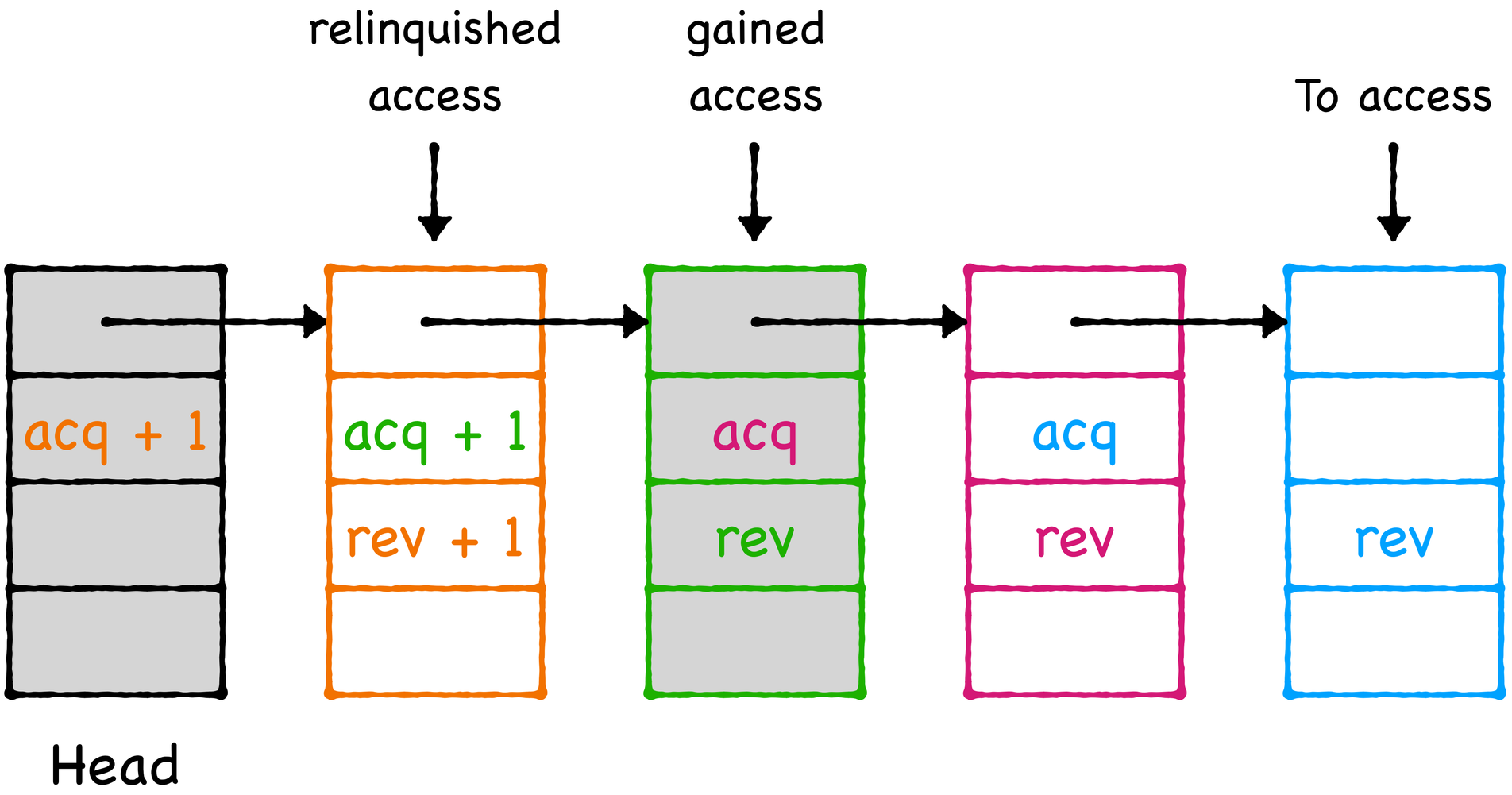

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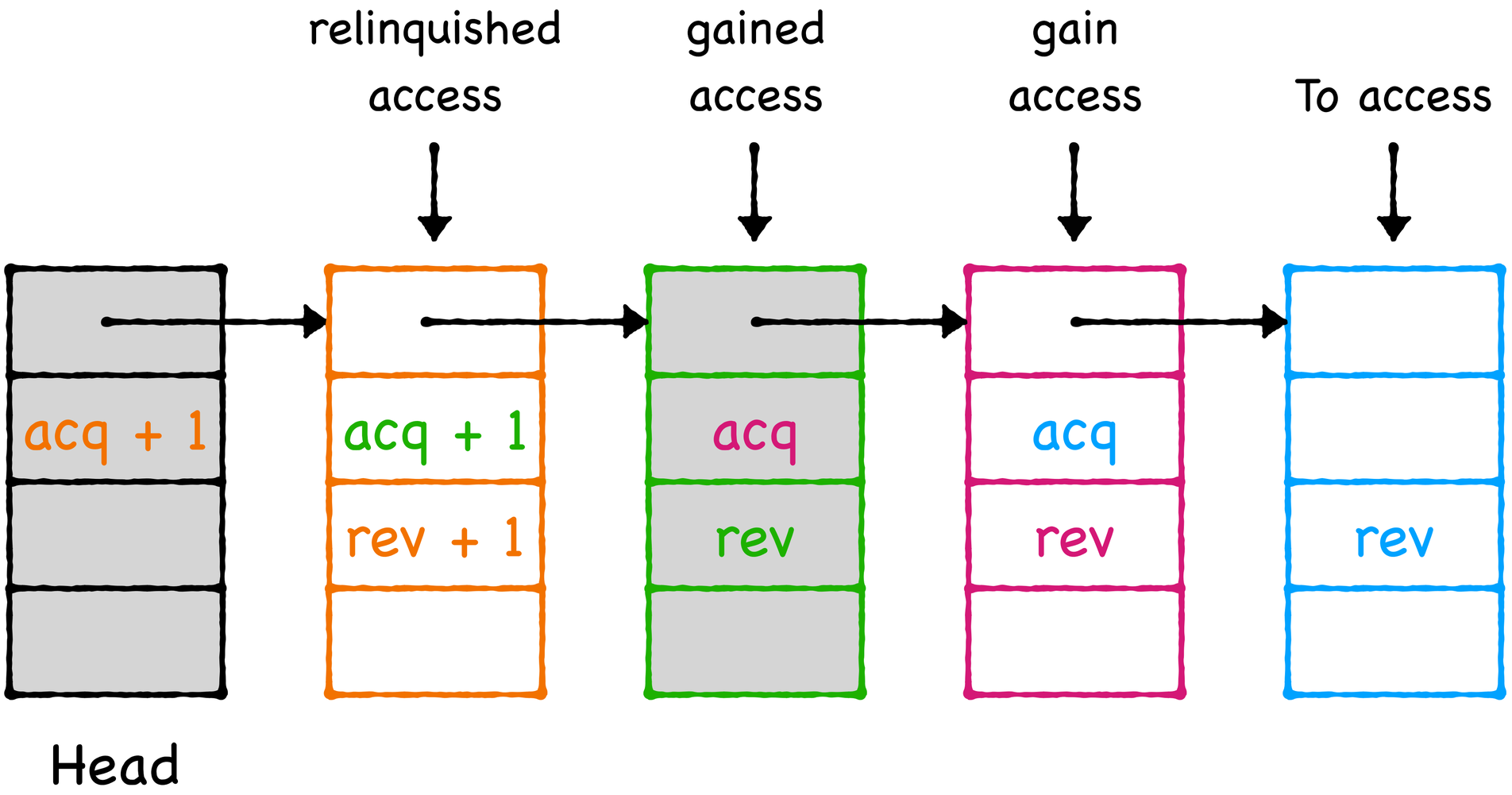

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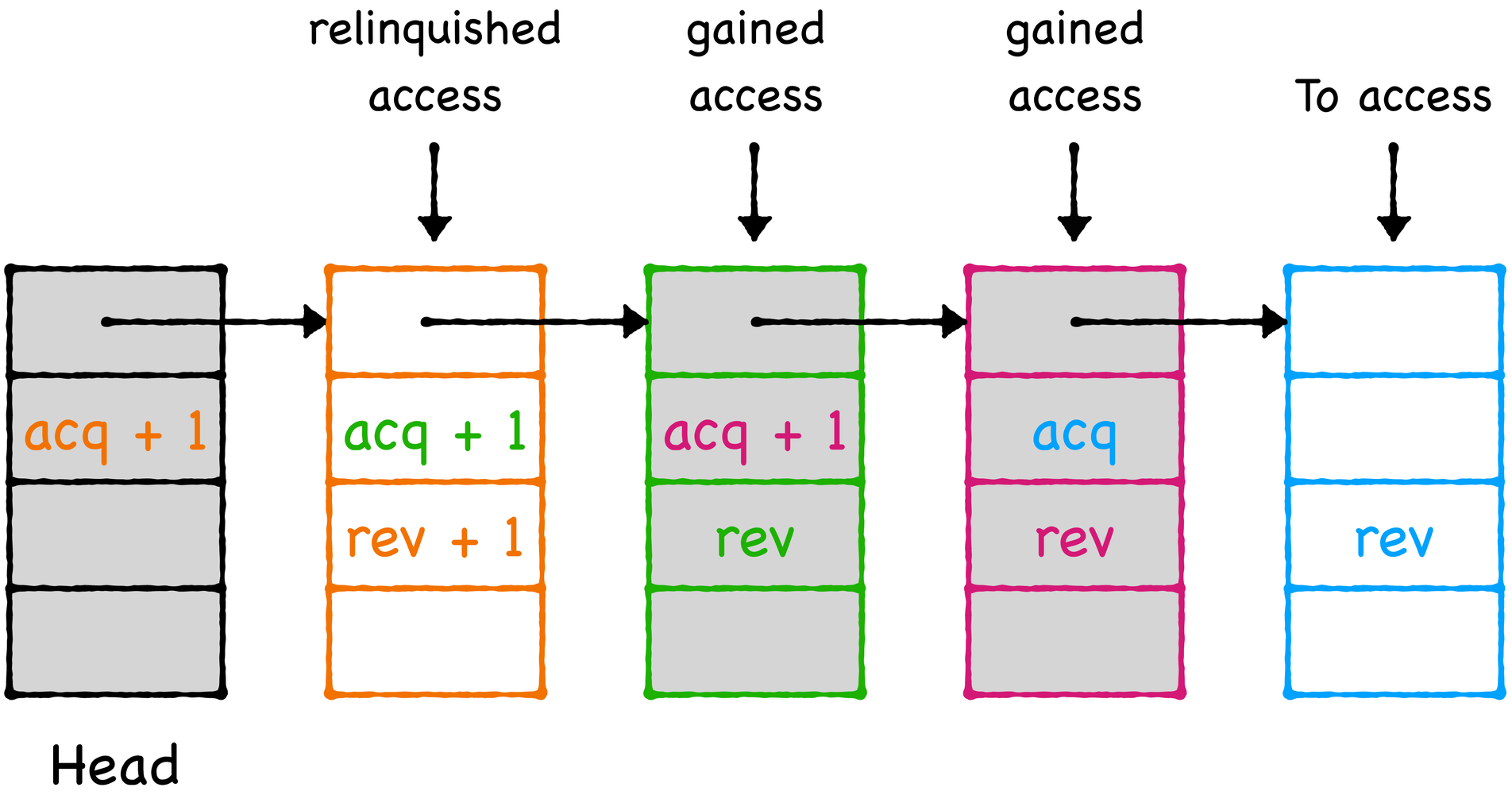

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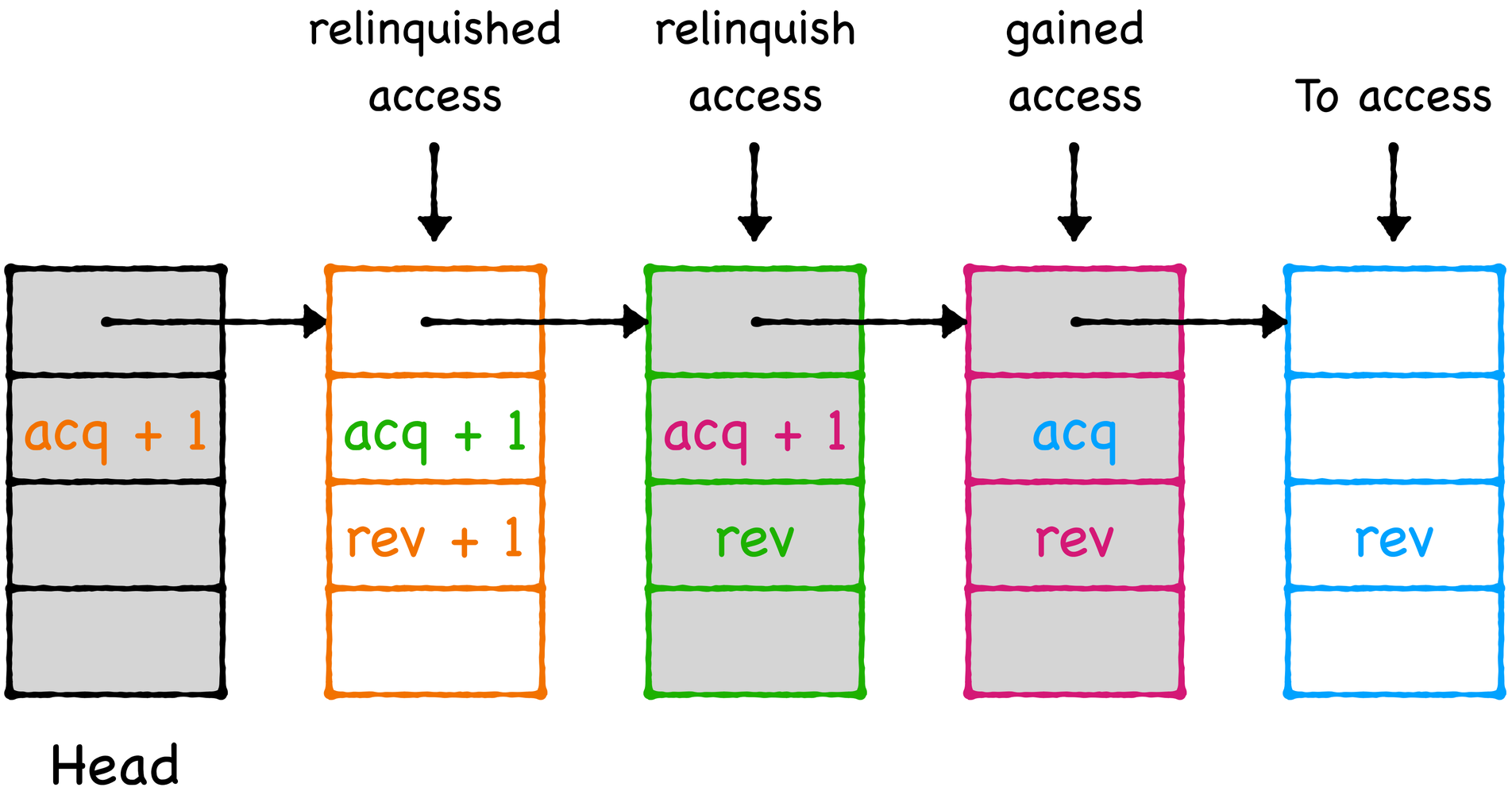

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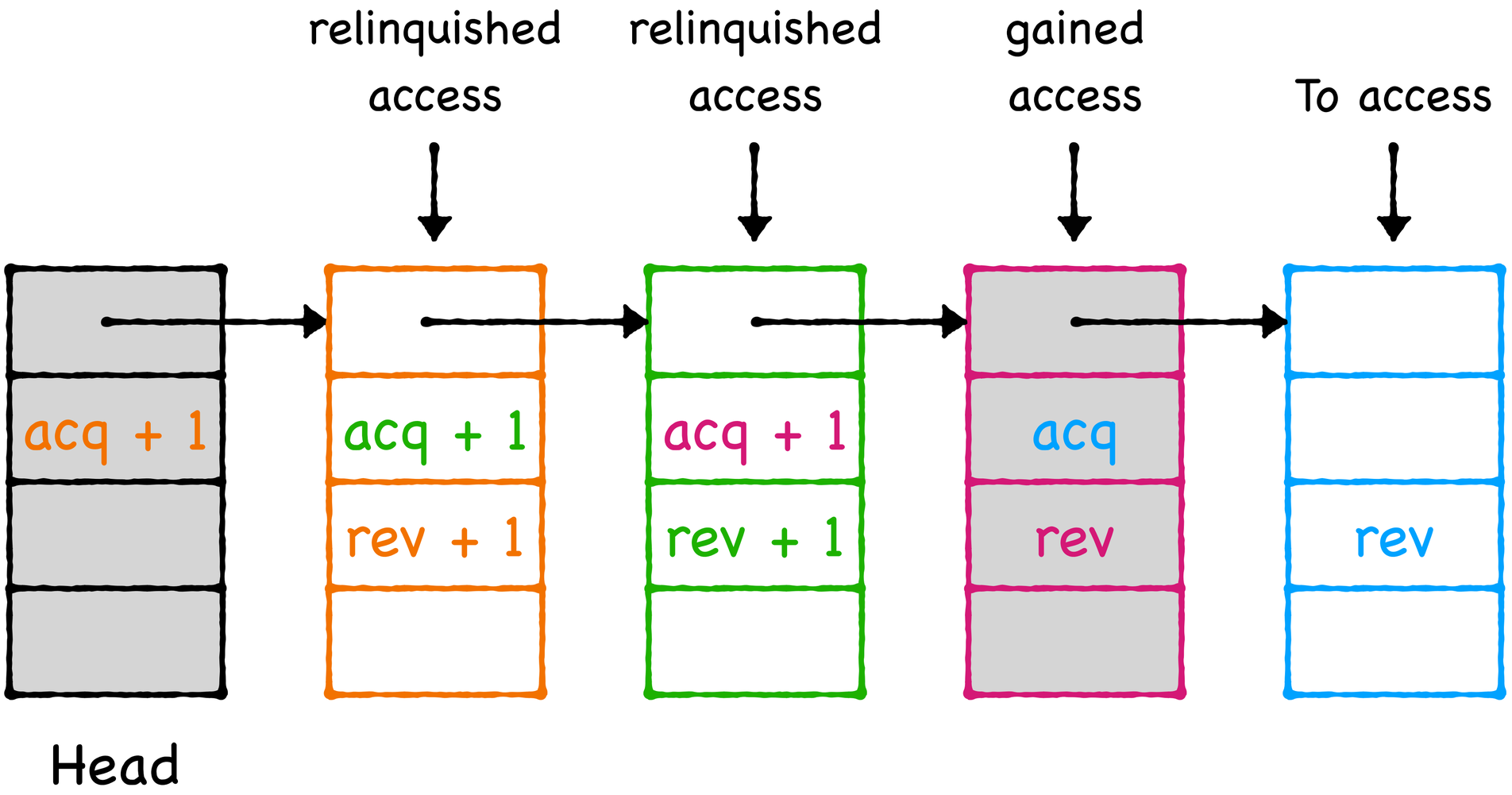

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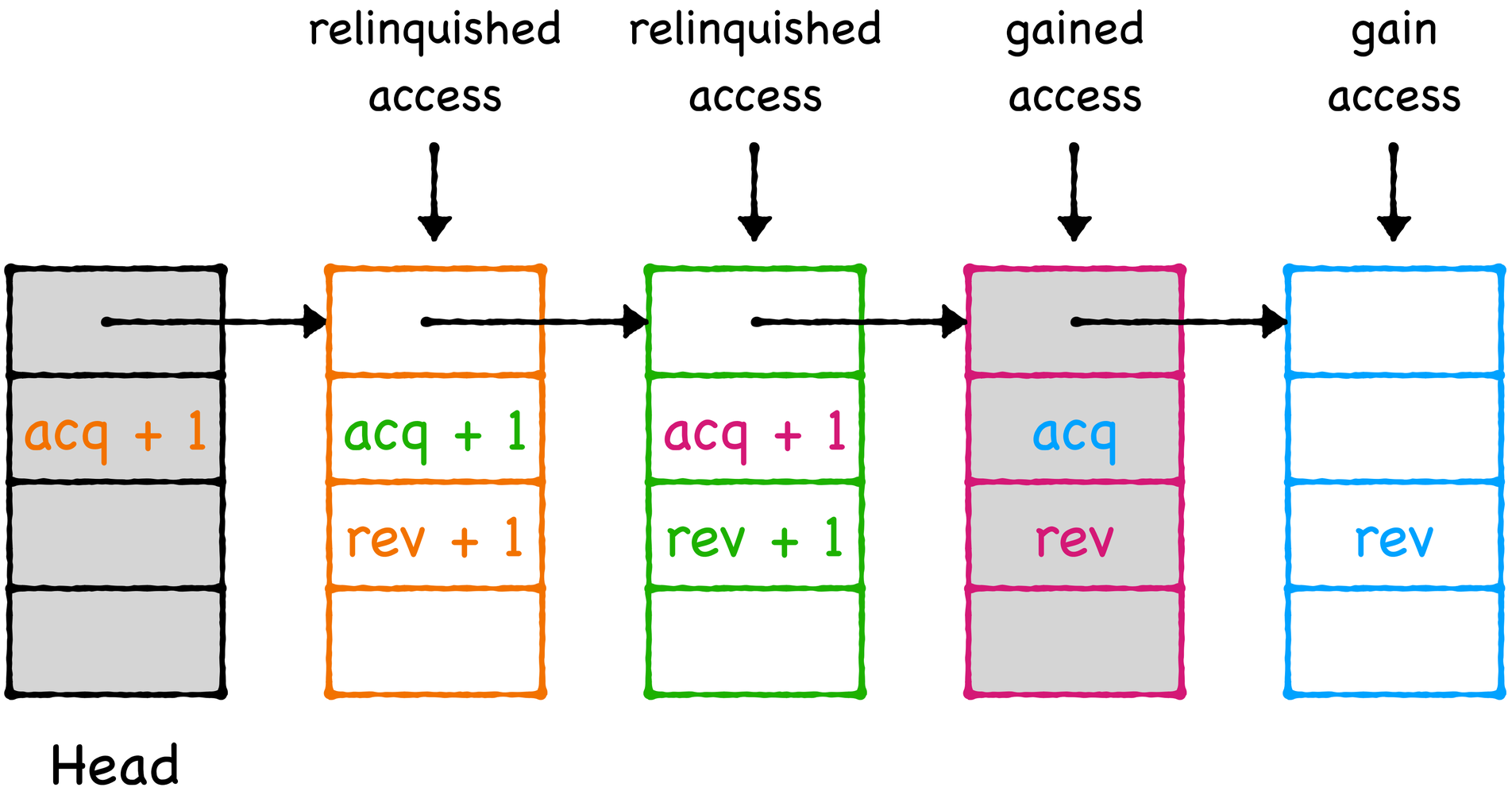

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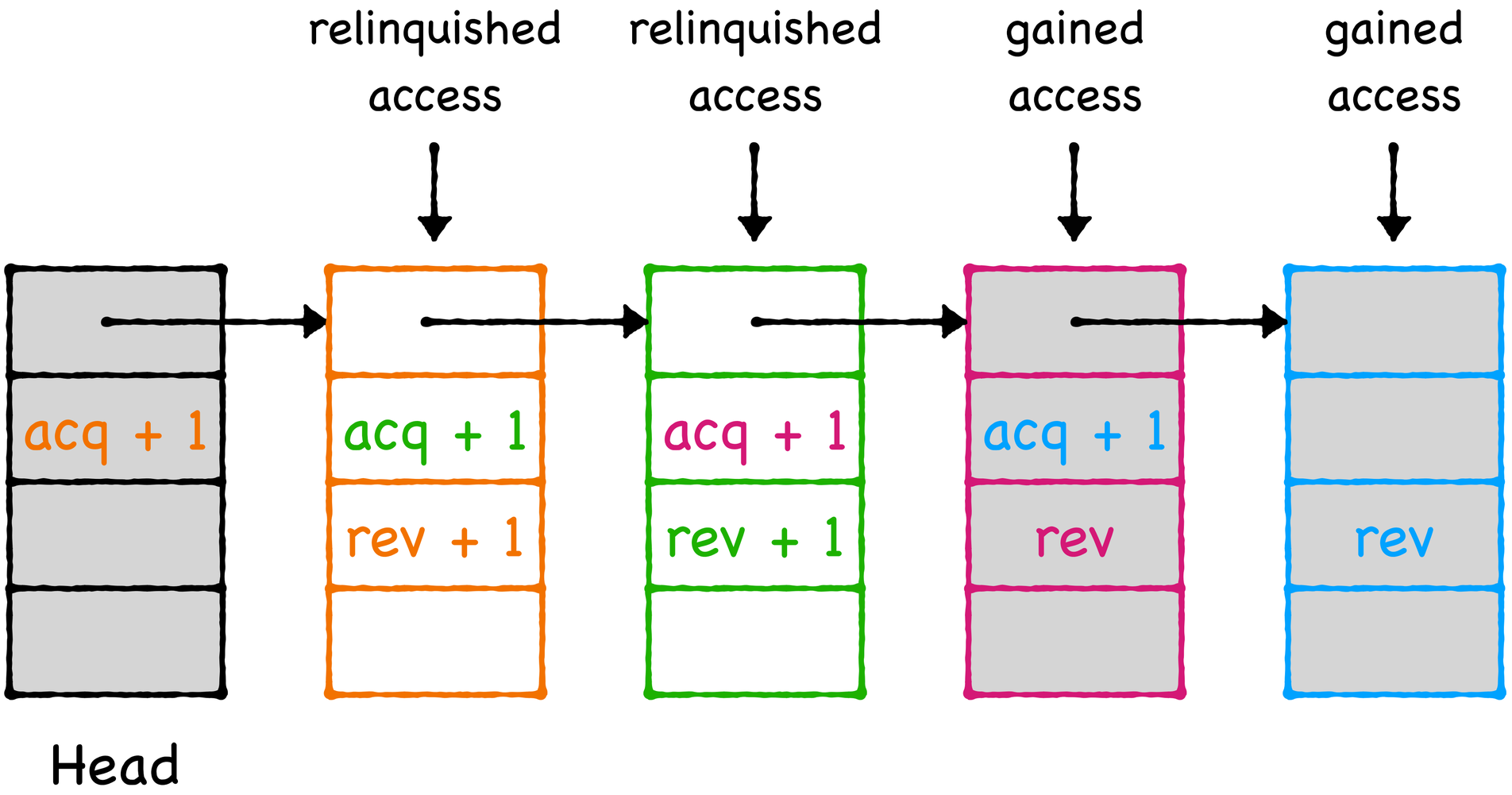

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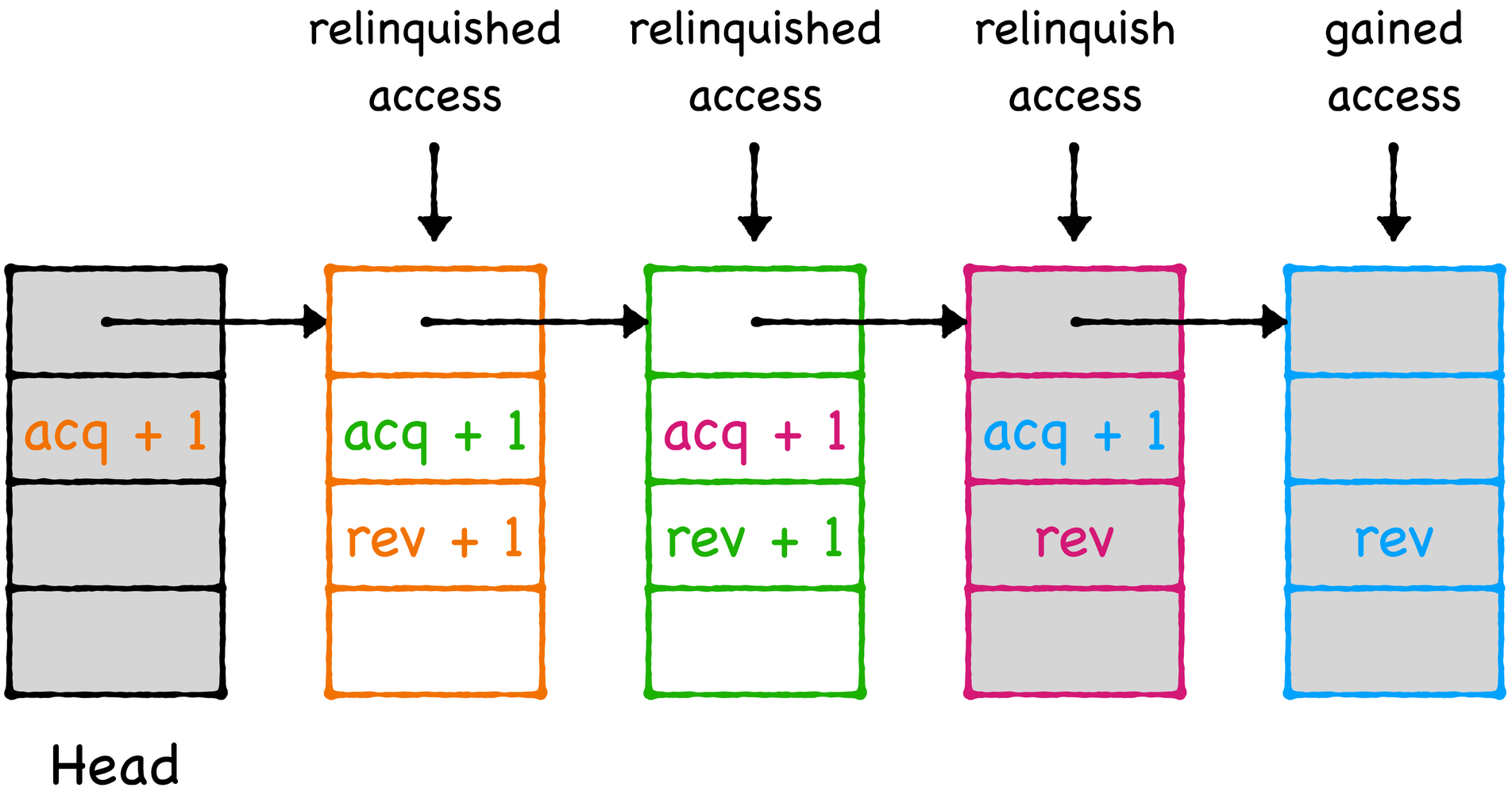

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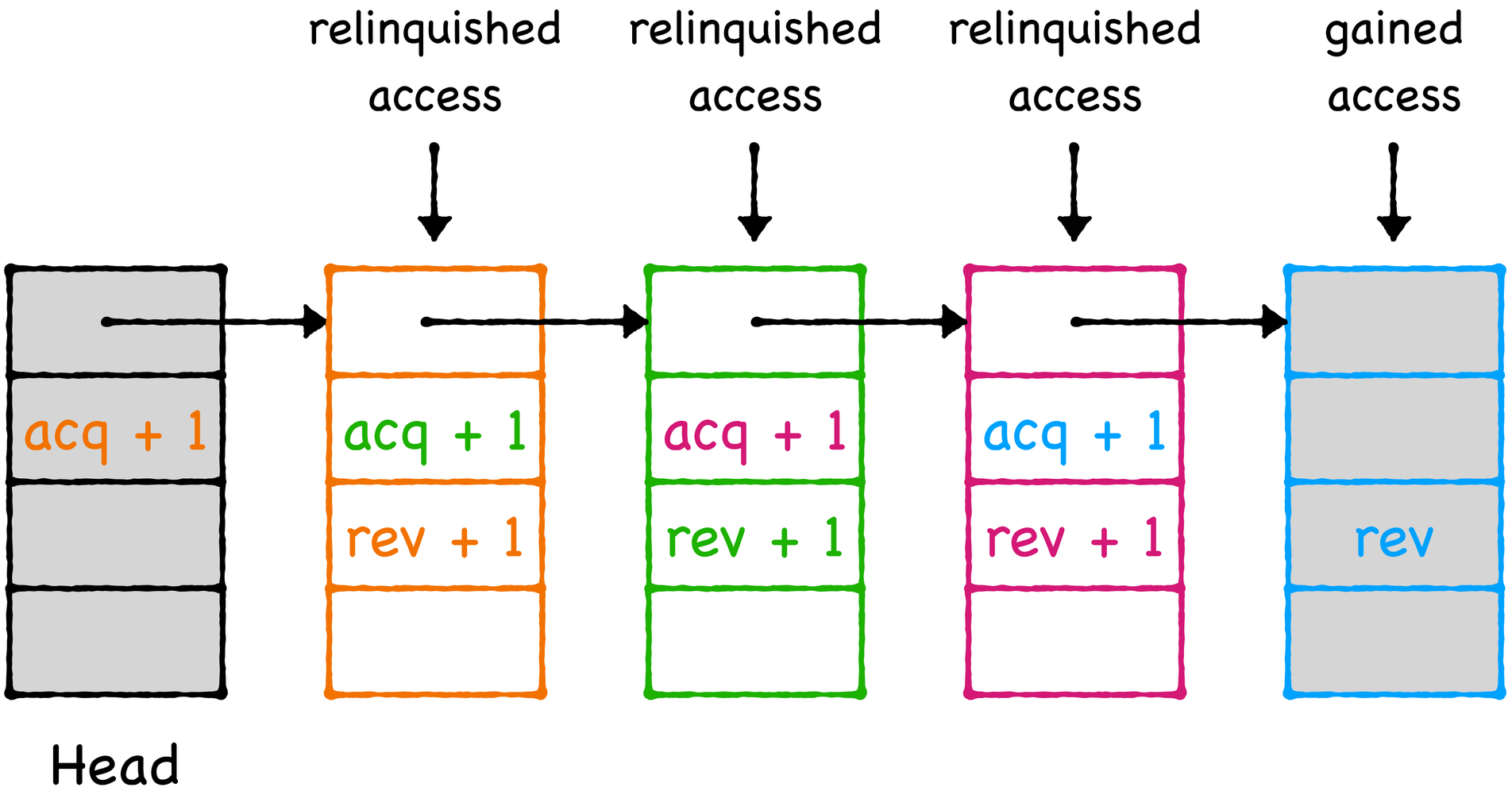

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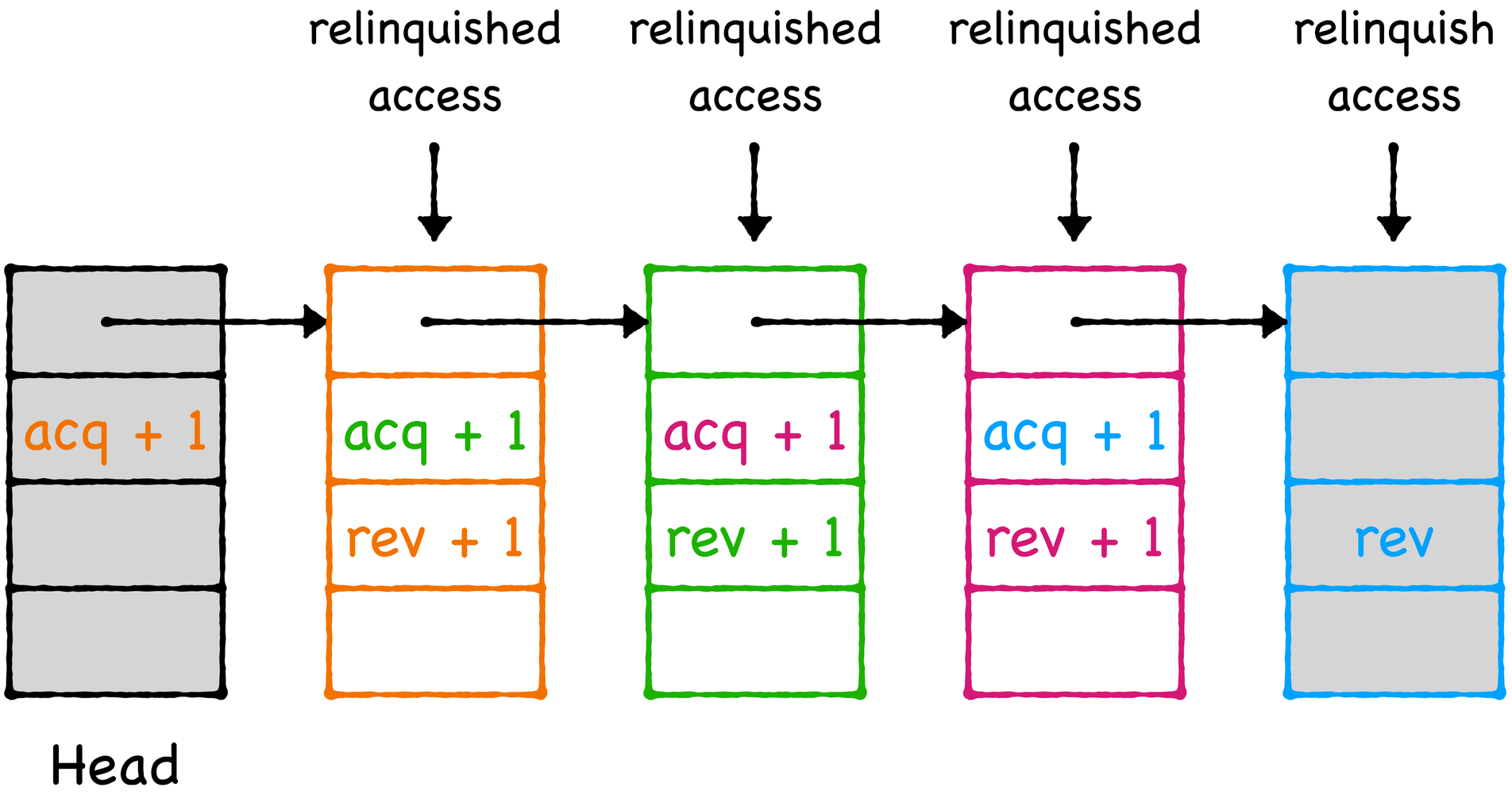

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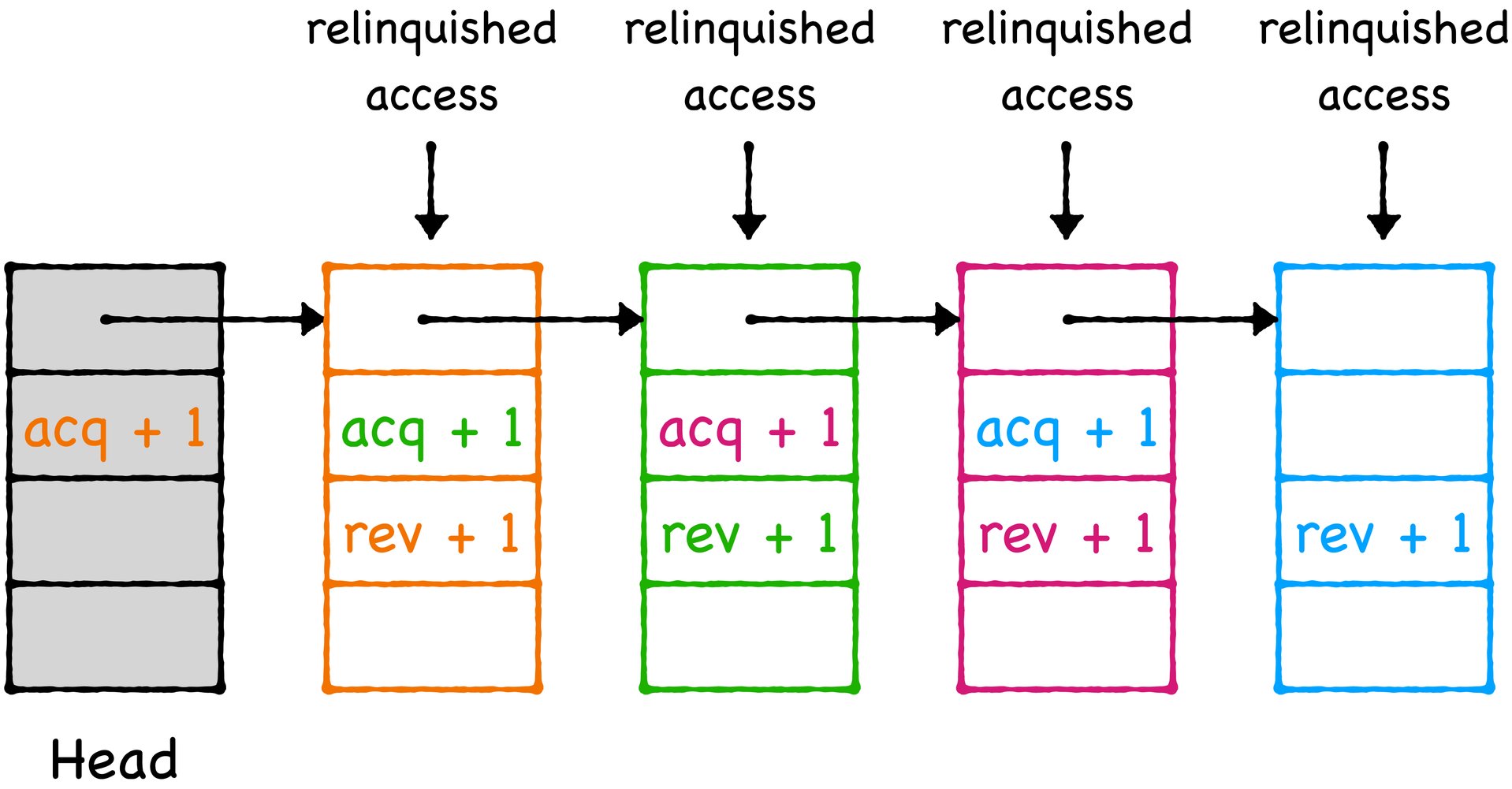

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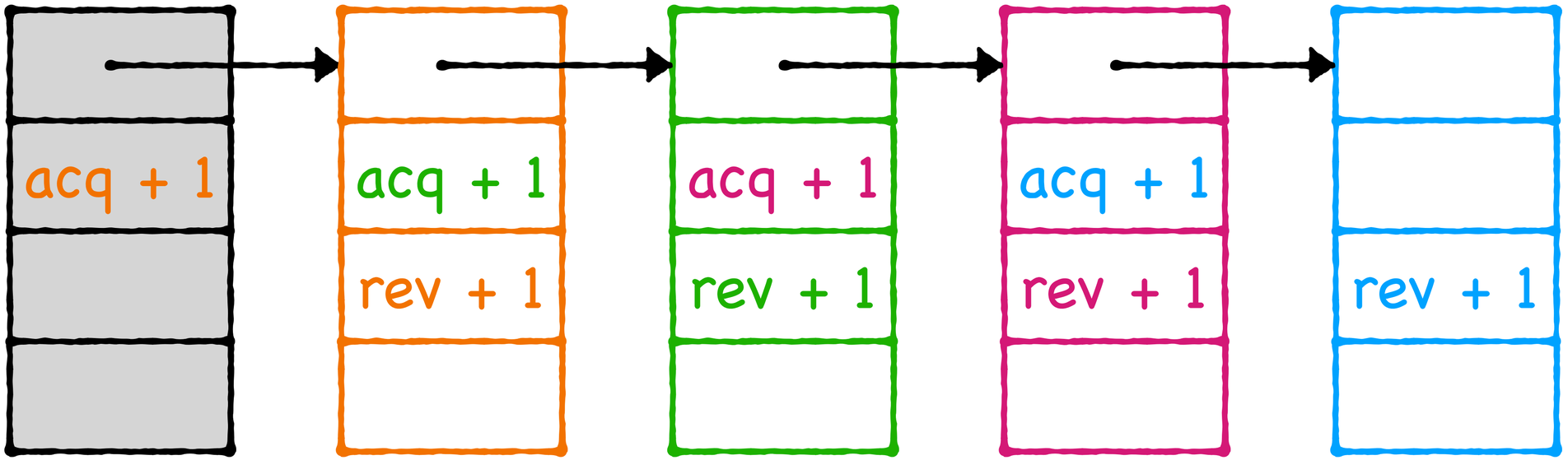
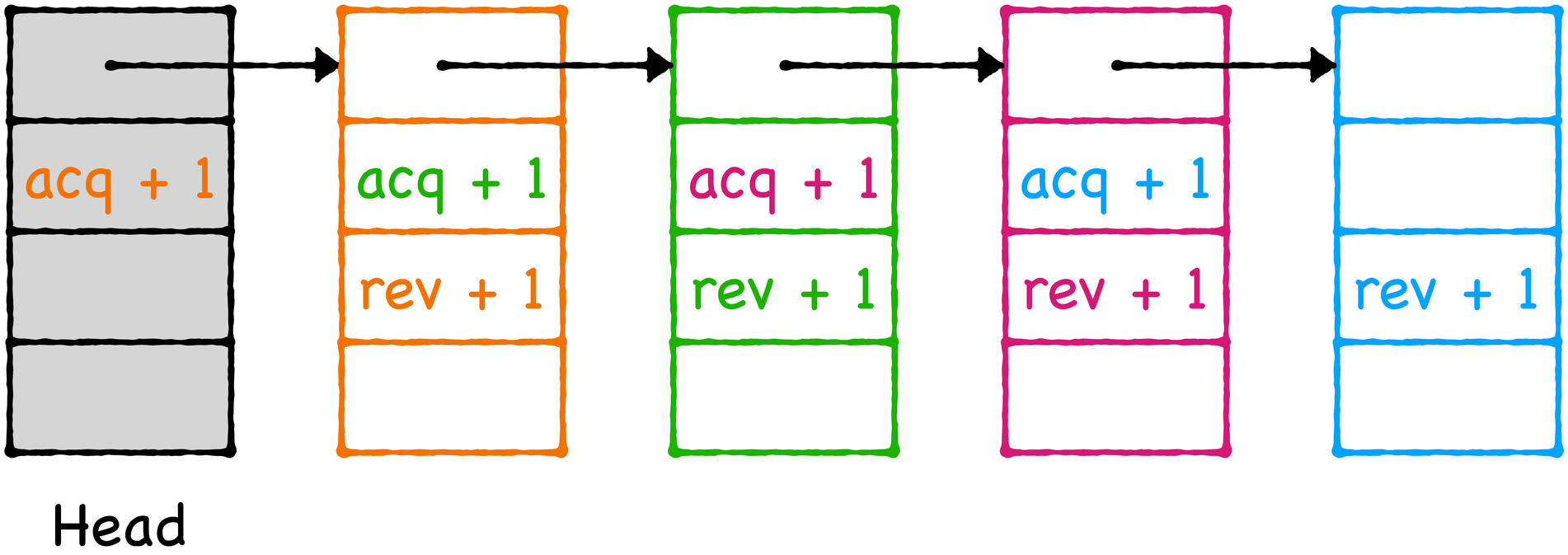
We can gain access to any cell in any list while having access to at most three cells simultaneously!



Space complexity O(c), i.e, linear in the point contention c



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  - the acquisition counter is shifted to the left
- But how do we compute ref = acq - rev atomically?

Our Second Universal Construction

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- We merged our recycling scheme with our first construction
- This merging was not easy!
- In the paper we explain the challenges and how we solved them

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(a) Space linear in the # of processes that have participated so far
Time linear in the point contention

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(3) A novel memory recycling scheme that may be of more general use

Open Questions

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(2) Can one achieve the space complexity of our second universal construction **without assuming bounded concurrency?**

(3) Are there other uses for our memory recycling scheme? In particular, the splitting of the reference counter?

(4) What other applications can benefit from GCAS?

Questions?